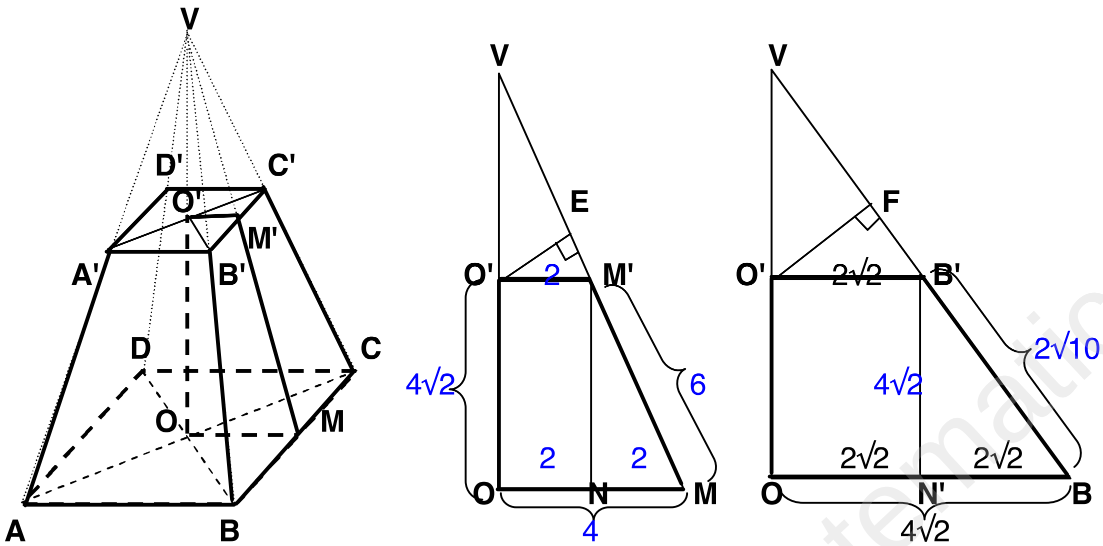
J1. TRUNCHIUL DE PIRAMIDĂ PATRULATERA REGULATA - PROBLEME REZOLVATE

1. Un trunchi de piramida patrulatera regulata are fata laterala un trapez isoscel ortodiagonal.Diagonalele bazelor au $8\sqrt{2}$ respectiv $4\sqrt{2}$cm Se cere: a) Al , At , V ; b) aria laterala si volumul piramidei din care provine trunchiul ; c) distanta de la centrul bazei mici la o fata laterala si distanta la o muchie laterala; d) Fie un punct P situat pe inaltimea trunchiului la egala distanta de planul bazei mari si planul unei fete laterale. Determinati pozitia punctului P fata de baza mare.



a) $Al = \dfrac{(P_B + P_b)\cdot a_{tr}}{2}$; $At = Al + A_B + A_b$; $V = \dfrac{h}{3}(A_B + A_b + \sqrt{A_B \cdot A_b})$

$AC = 8\sqrt{2}$, dar $AC = AB\sqrt{2} \Rightarrow AB\sqrt{2} = 8\sqrt{2} \Rightarrow$ **AB=8cm** ; $A'C' = 4\sqrt{2} \Rightarrow A'B'\sqrt{2} = 4\sqrt{2} \Rightarrow$ **A'B'=4cm**

Daca trapezul isoscel BCC'B' este ortodiagonal $\Rightarrow MM' = \dfrac{BC + B'C'}{2} = \dfrac{8+4}{2} = \dfrac{12}{2} \Rightarrow$ **MM'=6cm**

Daca AB = 8 cm $\Rightarrow$ **OM = 4 cm** ; A'B' = 4 cm $\Rightarrow$ **O'M' = 2 cm** ; $AC = BD = 8\sqrt{2} \Rightarrow$ **$OB = 4\sqrt{2}$ cm**

Analog **$O'B' = 2\sqrt{2}$ cm**

In trapezul O'M'MO $\Rightarrow$ NM = OM - O'M' = 4 - 2 = 2 cm.

In $\Delta M'NM, m\angle N = 90° \Rightarrow M'N^2 = M'M^2 - NM^2 = 36 - 4 = 32 \Rightarrow M'N = \sqrt{32} \Rightarrow$ **$M'N = 4\sqrt{2}$cm** $\Rightarrow$ **$O'O = 4\sqrt{2}$cm**

In$\Delta B'N'B$, $\angle N' = 90° \Rightarrow B'B^2 = B'N'^2 + N'B^2 = 32 + 8 = 40 \Rightarrow B'B = \sqrt{40} \Rightarrow$ **$B'B = 2\sqrt{10}$cm**

Al$= \dfrac{(32+16)\cdot 6}{2} =$ **144cm²** ; **At**=144 + 64 + 16 = **224cm²** ; **V**$= \dfrac{4\sqrt{2}}{3}(64 + 16 + 8\cdot 4) =$ **$\dfrac{448\sqrt{2}}{3}$ cm³** .

b) $O'M' \parallel OM \Rightarrow \Delta VO'M' \sim \Delta VOM \Rightarrow \dfrac{VO'}{VO} = \dfrac{O'M'}{OM} = \dfrac{VM'}{VM} \Rightarrow \dfrac{VO - 4\sqrt{2}}{VO} = \dfrac{2}{4} = \dfrac{VM - 6}{VM} \Rightarrow$

$4(VO - 4\sqrt{2}) = 2VO \Rightarrow 4VO - 2VO = 16\sqrt{2} \Rightarrow 2\cdot VO = 16\sqrt{2} \Rightarrow$ **$VO = 8\sqrt{2}$ cm**

$4(VM - 6) = 2VM \Rightarrow 4VM - 2VM = 24 \Rightarrow 2VM = 24 \Rightarrow$ **VM=12cm**

$Al_{PIRAMIDA} = \frac{P_B \cdot VM}{2} = \frac{32\cdot 12}{2} = 192 cm^2$; $V_{PIRAMIDA} = \frac{A_B \cdot VO}{2} = \frac{64\cdot 8\sqrt{2}}{2} = 256\sqrt{2}\ cm^3$.

c) **d(O'; (BCC'B'))** = d(O'; (VBC)) = d(O'; VM) = **O'E** (O'E ⊥ VM);

ΔVEO' ~ ΔVOM (∠E = ∠O = 90°;∠V-comun) ⇒

$$\frac{O'E}{OM} = \frac{VO'}{VM} \Rightarrow \frac{O'E}{4} = \frac{4\sqrt{2}}{12} \Rightarrow O'E = \frac{4\cdot 4\sqrt{2}}{12} \Rightarrow O'E = \frac{4\sqrt{2}}{3}\ cm .$$

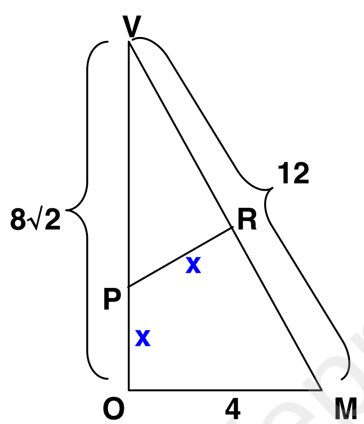
In ΔVOB,∠O=90° ⇒ $VB^2 = VO^2 + OB^2 = 128 + 32 = 160 \Rightarrow VB = \sqrt{160} \Rightarrow$ **VB=4√10cm.**

d(O';(BCC'B')) = d(O'; VB) = **O'F** (O'F⊥VB)

ΔVFO' ~ ΔVOB (∠F = ∠O = 90°; ∠V-comun) ⇒

$$\frac{O'F}{OB} = \frac{VO'}{VB} \Rightarrow \frac{O'F}{4\sqrt{2}} = \frac{4\sqrt{2}}{4\sqrt{10}} \Rightarrow O'F = \frac{4\sqrt{2}\cdot 4\sqrt{2}}{4\sqrt{10}} \Rightarrow O'F = \frac{4\sqrt{10}}{5}\ cm$$

d)



Notez **PO = PR = x**

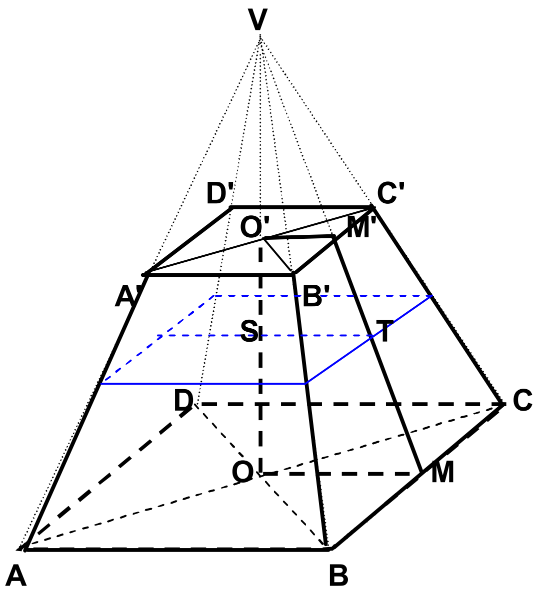
ΔVRP ~ ΔVOM (drept. cu V unghi comun) ⇒ $\frac{PR}{OM} = \frac{VP}{VM} \Rightarrow \frac{x}{4} = \frac{8\sqrt{2} - x}{12} \Rightarrow$

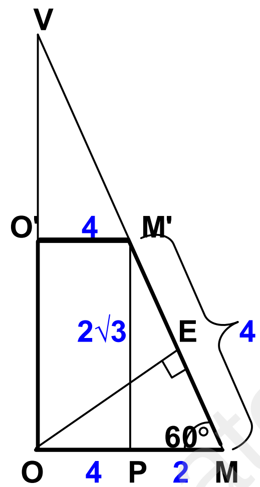
⇒ 12·x = 4·(8√2 - x) ⇒ 12x = 32√2 - 4x ⇒ 12x + 4x = 32√2 ⇒ 16x = 32√2 ⇒ **x = 2√2 cm** ⇒

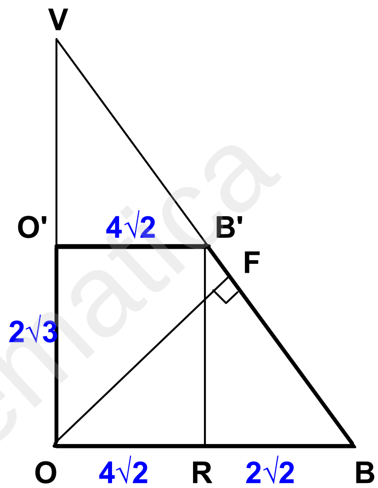
PO = 2√2 cm

2. Un trunchi de piramida patrulatera regulata ABCDA'B'C'D' are unghiul dintre o fața laterala si planul bazei mari de 60°, perimetrul bazei mici 32 cm si aria laterala 160 cm^2. Se cere:
a) Aria totala si volumul trunchiului de piramida.
b) Aria laterala si volumul piramidei in care provine trunchiul.
c) Distantele de la centrul bazei mari la o fața laterala si la o muchie laterala a trunchiului.
d) La ce distanta fata de baza mica trebuie facuta o sectiune paralela cu bazele, astfel incat aria sectiunii obtinute sa fie egala cu 100 cm^2.
e) Distantele de la punctul B' respectiv de la punctul B la planul (ADD')

REZOLVARE







a) Perimetrul bazei mici = 4·l ⇒ 4·l = 32 ⇒ **l = 8 ⇒ A'B' = 8 cm**

$O'M' = \frac{l}{2} = \frac{A'B'}{2} = \frac{8}{2} = 4 \Rightarrow$ **O'M' = 4cm** ; $O'B' = \frac{l\sqrt{2}}{2} = \frac{AB\sqrt{2}}{2} = \frac{8\sqrt{2}}{2} = 4\sqrt{2} \Rightarrow$ **O'B' = 4√2 cm**

$\angle((BCC'B') ; (ABCD)) = \angle(MM'; OM) = \angle(M'MO) = 60°$

In ΔM'PM dr. in P, daca $m\angle(M'MP) = 60° \Rightarrow m\angle(MM'P) = 30° \Rightarrow PM = \frac{MM'}{2} \Rightarrow MM' = 2 \cdot PM$

Notez PM = x ⇒ MM'= 2x . Perimetrul bazei mari = $P_B = 4 \cdot AB$, dar $AB = 2 \cdot OM \Rightarrow P_B = 8 \cdot OM$

Deoarece OM = OP + PM ⇒ OM = 4 + x ⇒ $P_B = 8 \cdot (4 + x) = 32 + 8x$

Perimetrul bazei mici = P_b = 32 cm

Aria laterala a trunchiului = $\frac{(P_B + P_b) \cdot \text{apotema trunchiului}}{2}$

$160 = \frac{(32 + 8x + 32) \cdot 2x}{2} \Rightarrow (64 + 8x) \cdot x = 160 \Rightarrow 8x^2 + 64x - 160 = 0 \ / :8 \Rightarrow x^2 + 8x - 20 = 0 \Rightarrow$

$\Rightarrow x^2 + 10x - 2x - 20 = 0 \Rightarrow x(x + 10) - 2(x + 10) = 0 \Rightarrow (x + 10) \cdot (x - 2) = 0 \Rightarrow$

$\Rightarrow x + 10 = 0 \Rightarrow$ **x = -10** ; $x - 2 = 0 \Rightarrow$ **x = 2**

Deoarece lucram cu segmente solutia valablila este **x = 2** ⇒ **PM = 2cm** iar **MM' = 4 cm**

In $\Delta M'PM$ dr. in P $\Rightarrow M'P^2 = M'M^2 - PM^2 \Rightarrow M'P^2 = 4^2 - 2^2 = 16 - 4 = 12 \Rightarrow M'P = \sqrt{12} \Rightarrow$ **$M'P = 2\sqrt{3}$ cm**

Daca $M'P = 2\sqrt{3}$ cm $\Rightarrow$ **$O'O = 2\sqrt{3}$ cm** ; Daca $PM = 2$ cm $\Rightarrow OM = 6$ cm $\Rightarrow$ **$AB = 12$ cm**

Deci elementele trunchiului sunt: **$l = 8$ cm , $L = 12$ cm , $a_{tr} = 4$ cm , $h_{tr} = 2\sqrt{3}$ cm.**

Aria totala = Aria laterala + Aria bazei mari + Aria bazei mici

Aria bazei mari = $A_B = L^2 = 12^2 = 144$ cm^2 ; Aria bazei mici = $A_b = l^2 = 8^2 = 64$ cm^2

Aria totala = $160 + 144 + 64 = 368$ cm^2 .

Volumul trunchiului $= \frac{h}{3} \cdot (A_B + A_b + \sqrt{A_B \cdot A_b}) = \frac{2\sqrt{3}}{3} \cdot (144 + 64 + \sqrt{144 \cdot 64}) = \frac{2\sqrt{3}}{3} \cdot (208 + 96)$

Volumul $= \frac{2\sqrt{3}}{3} \cdot 304 \Rightarrow$ **Volumul trunchiului = $\frac{608\sqrt{3}}{3}$ cm^3 .**

b) $O'M' \parallel OM \Rightarrow \Delta VO'M' \sim \Delta VOM \Rightarrow \frac{VO'}{VO} = \frac{O'M'}{OM} = \frac{VM'}{VM} \Rightarrow \frac{VO - 2\sqrt{3}}{VO} = \frac{4}{6} = \frac{VM - 4}{VM} \Rightarrow$

$6 \cdot (VO - 2\sqrt{3}) = 4VO \Rightarrow 6 \cdot VO - 4 \cdot VO = 12\sqrt{3} \Rightarrow 2 \cdot VO = 12\sqrt{3} \Rightarrow$ **$VO = 6\sqrt{3}$ cm**

$6 \cdot (VM - 4) = 4 \cdot VM \Rightarrow 6 \cdot VM - 4VM = 24 \Rightarrow 2 \cdot VM = 24 \Rightarrow$ **$VM = 12$ cm .**

Aria laterala a piramidei $= \frac{P_B \cdot VM}{2} = \frac{4 \cdot 12 \cdot 12}{2} =$ **288 cm^2** .

Volumul piramidei $= \frac{A_B \cdot VO}{3} = \frac{144 \cdot 6\sqrt{3}}{3} =$ **$288\sqrt{3}$ cm^3** .

c) d(O ; (BCC'B'))

$$\left.\begin{array}{l} M'M \perp BC \\ OM \perp BC \\ OM, M'M \subset (OMM'O') \end{array}\right\} \Rightarrow BC \perp (OMM'O')$$

Construiesc $OE \perp M'M$. Deoarece $OE \subset (OMM'O')$ si $BC \perp (OMM'O') \Rightarrow BC \perp OE \Rightarrow OE \perp BC$

Din $OE \perp M'M$ si $OE \perp BC \Rightarrow OE \perp (BCC'B') \Rightarrow$ **d(O; (BCC'B') = OE**

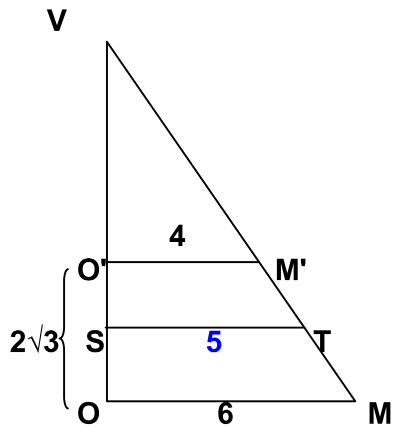
In ΔVOM dr. in O $\Rightarrow OE = \frac{VO \cdot OM}{VM} = \frac{6\sqrt{3} \cdot 6}{12} \Rightarrow$ **$OE = 3\sqrt{3}$ cm**

d(O; VB) . Construiesc $OF \perp VB \Rightarrow$ **d(O; VB) = OF**

In ΔVOB dr. in O $\Rightarrow OF = \frac{VO \cdot OB}{VB} = \frac{6\sqrt{3} \cdot 6\sqrt{2}}{6\sqrt{5}} = \frac{6\sqrt{6}}{\sqrt{5}} \Rightarrow$ **$OF = \frac{6\sqrt{30}}{5}$ cm .**

In ΔVOB dr. $\Rightarrow VB^2 = VO^2 + OB^2 \Rightarrow VB^2 = (6\sqrt{3})^2 + (6\sqrt{2})^2 = 108 + 72 = 180 \Rightarrow$ **$VB = 6\sqrt{5}$ cm.**

d)



Deoarece sectiunea este un patrat cu aria 100 cm^2 ⇒

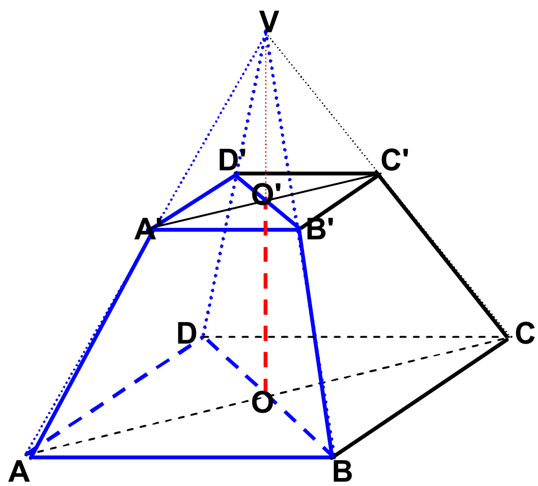
latura sectiunii este $l' = \sqrt{100} \Rightarrow l' = 10$ cm

Daca l' = 10 cm ⇒ $ST = \frac{l'}{2} = \frac{10}{2} \Rightarrow$ **ST = 5cm**

In trapezul OMM'O' se observa ca $ST = \frac{O'M' + OM}{2} \Rightarrow$

⇒ ST este linie mijlocie in trapez ⇒ **$SO' = SO = \sqrt{3}$ cm**

e) d(B'; ADD') ; d(B ; ADD')



Planul (ADD') este inclus in planul feței laterale a trunchiului de piramida, care este inclus in planul feței laterale a piramidei din care provine trunchiul.

d(B'; ADD') este echivalenta cu d(B';VA'D')

d(B; ADD') este echivalenta cu d(B;VAD)

d(B'; VA'D') ⇒ formez piramida B'VA'D' si scriu volumul ei in doua moduri.

$$V_{B'VA'D'} = \frac{\text{Aria } \Delta VA'D' \cdot d(B';VA'D')}{3} = \frac{\text{Aria } \Delta A'B'D' \cdot VO'}{3} \Rightarrow \mathbf{d(B';VA'D') = \frac{Aria\ \Delta A'B'D' \cdot VO'}{Aria\ \Delta VA'D'}}$$

$$\text{Aria } \Delta A'B'D' = \frac{l^2}{2} = \frac{64}{2} = 32\ cm^2. \quad \text{Aria } \Delta VA'D' = \text{Aria } \Delta VB'C' = \frac{VM' \cdot B'C'}{2} = \frac{8 \cdot 8}{2} = 32\ cm^2$$

$$d(B'; VA'D') = \frac{32 \cdot 4\sqrt{3}}{32} \Rightarrow \mathbf{d(B'; VA'D') = 4\sqrt{3}\ cm}$$

d(B; VAD) ⇒ formez piramida BVAD si scriu volumul ei in doua moduri.

$$V_{B'VA'D'} = \frac{\text{Aria } \Delta VAD \cdot d(B ;VAD)}{3} = \frac{\text{Aria } \Delta ABD \cdot VO}{3} \Rightarrow \mathbf{d(B; VAD) = \frac{Aria\ \Delta ABD \cdot VO}{Aria\ \Delta VAD}}$$

$$\text{Aria } \Delta ABD = \frac{L^2}{2} = \frac{144}{2} = 72\ cm^2. \quad \text{Aria } \Delta VAD = \text{Aria } \Delta VBC = \frac{VM \cdot BC}{2} = \frac{12 \cdot 12}{2} = 72\ cm^2$$

$$d(B; VAD) = \frac{72 \cdot 6\sqrt{3}}{72} \Rightarrow \mathbf{d(B; VAD) = 6\sqrt{3}\ cm}$$