L3. FUNCTIA DE GRADUL I - PROBLEME REZOLVATE

5. Fie funcțiile $f:\mathbb{R}\to\mathbb{R}$, $f(x)=-\sqrt{3}x+2$; $g:\mathbb{R}\to\mathbb{R}$, $g(x)=\sqrt{3}x+2$.
a) Aflati coordonatele punctului de intersectie a graficelor functiilor f si g
b) Aflati unghiul dintre graficele functiilor f si g
c) Aflati aria suprafetei dintre graficele celor 2 functii si abscisa
d) Aflati coordonatele unui punct de pe graficul functiei f in care ordonata este dublul abscisei

REZOLVARE

a) Pentru a afla coordonatele punctului de intersectie a graficelor a 2 functii , se egaleaza cele 2 functii, se afla **x** , apoi se inlocuieste **x** in **f(x)** si se afla si valoarea lui **f(x)**. Punctul de intersectie are coordonatele **I(x ; f(x))**

$$\left.\begin{array}{l} -\sqrt{3}\cdot x + 2 = \sqrt{3}\cdot x + 2 \Rightarrow -2\sqrt{3}\cdot x = 0 \Rightarrow \mathbf{x = 0} \\ f(0) = -\sqrt{3}\cdot 0 + 2 \Rightarrow \mathbf{f(0) = 2} \end{array}\right\} \Rightarrow \mathbf{I(0\ ;\ 2)}$$

b) Pentru trasarea graficului deoarece trebuie sa aflam unghiul dintre graficele functiilor, aflam coordonatele punctelor in care graficele functiilor intersecteaza axele (facem intersectia cu axele)

$x=0 \Rightarrow \mathbf{f(0)=2}$
$f(x)=0 \Rightarrow -\sqrt{3}\cdot x+2=0 \Rightarrow \mathbf{x = \frac{2\sqrt{3}}{3}} \Rightarrow \mathbf{A\left(\frac{2\sqrt{3}}{3}\ ;\ 0\right)}$

$x=0 \Rightarrow \mathbf{g(0)=2}$
$g(x)=0 \Rightarrow \sqrt{3}\cdot x+2=0 \Rightarrow \mathbf{x = -\frac{2\sqrt{3}}{3}} \Rightarrow \mathbf{B\left(-\frac{2\sqrt{3}}{3}\ ;\ 0\right)}$

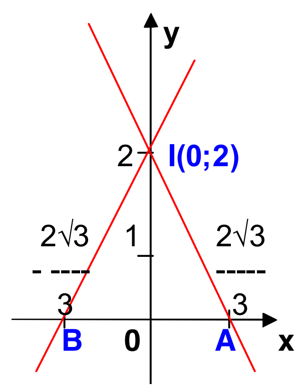
Pentru functia **f** am obtinut punctele $\mathbf{A\left(\frac{2\sqrt{3}}{3}\ ;\ 0\right)}$ si **I(0 ; 2)**

Pentru functia **g** am obtinut punctele $\mathbf{B\left(-\frac{2\sqrt{3}}{3}\ ;\ 0\right)}$ si **I(0 ; 2)**

In Δ IOA $\Rightarrow IA^2 = \left(\frac{2\sqrt{3}}{3}\right)^2 + 2^2 = \frac{12}{9} + 4 = \frac{48}{9} \Rightarrow IA = \frac{4\sqrt{3}}{3}$

$IB = IA = AB = \frac{4\sqrt{3}}{3} \Rightarrow \Delta IAB$ echilateral $\Rightarrow$ **m(∠ BIA) = 60°**

c) Aria ΔBIA $= \frac{AB\cdot OI}{2} = \frac{4\sqrt{3}}{3}\cdot\frac{2}{2} = \mathbf{\frac{4\sqrt{3}}{3}}$ (ua)



d) Orice punct de pe grafic are coordonatele **x** si **f(x)** unde **x = abscisa** si **f(x) = ordonata**

Daca ordonata este dublul abscisei $\Rightarrow$ **f(x) = 2·x** $\Rightarrow -\sqrt{3}x + 2 = 2\cdot x \Rightarrow -\sqrt{3}\cdot x - 2x = -2 \Rightarrow x = \frac{2}{2+\sqrt{3}} \Rightarrow$

$\Rightarrow x = \frac{2(2-\sqrt{3})}{4-3} \Rightarrow \mathbf{x=2(2-\sqrt{3})} \quad \Rightarrow \mathbf{f(2(2-\sqrt{3}))} = -\sqrt{3}\cdot 2(2-\sqrt{3})+2 = -4\sqrt{3}+6+2 = 8-4\sqrt{3} = \mathbf{4(2-\sqrt{3})}$

Deci coordonatele punctului sunt **M(2(2-√3) ; 4(2-√3))**

6. Fie functiile $f:\mathbb{R} \to \mathbb{R}$, $f(x) = ax + 6$ si $g:\mathbb{R}\to\mathbb{R}$, $g(x) = bx - 6$. Punctul de intersectie a graficelor celor doua functii este $I(3\sqrt{3}; 3)$.

a) Aratati ca: $a = -\dfrac{\sqrt{3}}{3}$ si $b = \sqrt{3}$.

b) Pentru valorile lui a si b de la punctul a) reprezentati in acelasi sistem de axe graficele celor doua functii.

c) Demonstrati ca, graficele celor 2 functii sunt perpendiculare.

d) Rezolvati in R inecutia: $3 \cdot f(m) \le g(m)$.

REZOLVARE

a) Daca $I(3\sqrt{3}; 3)$ este punct de intersectie a graficelor $\Rightarrow$ punctul apartine ambelor grafice $\Rightarrow$

$\mathbf{I(3\sqrt{3}; 3) \in G_f} \Rightarrow f(3\sqrt{3}) = 3$ dar $f(3\sqrt{3}) = 3\sqrt{3}\cdot a + 6 \Rightarrow 3\sqrt{3}\cdot a + 6 = 3 \Rightarrow 3\sqrt{3}\cdot a = -3 \Rightarrow \mathbf{a = -\dfrac{\sqrt{3}}{3}}$

$\mathbf{I(3\sqrt{3}; 3) \in G_g} \Rightarrow g(3\sqrt{3}) = 3$ dar $g(3\sqrt{3}) = 3\sqrt{3}\cdot b - 6 \Rightarrow 3\sqrt{3}\cdot b - 6 = 3 \Rightarrow 3\sqrt{3}\cdot b = 9 \Rightarrow \mathbf{b = \sqrt{3}}$

b) Daca $a = -\dfrac{\sqrt{3}}{3} \Rightarrow \mathbf{f(x) = -\dfrac{\sqrt{3}}{3}x + 6}$

Daca $b = \sqrt{3} \Rightarrow \mathbf{g(x) = \sqrt{3}x - 6}$

Intocmesc tabelele de valori

x	$-\infty$	**0**	$\mathbf{6\sqrt{3}}$	$+\infty$
f(x)		**6**	**0**	
		A	**B**	

x	$-\infty$	**0**	$\mathbf{2\sqrt{3}}$	$+\infty$
g(x)		**-6**	**0**	
		C	**D**	

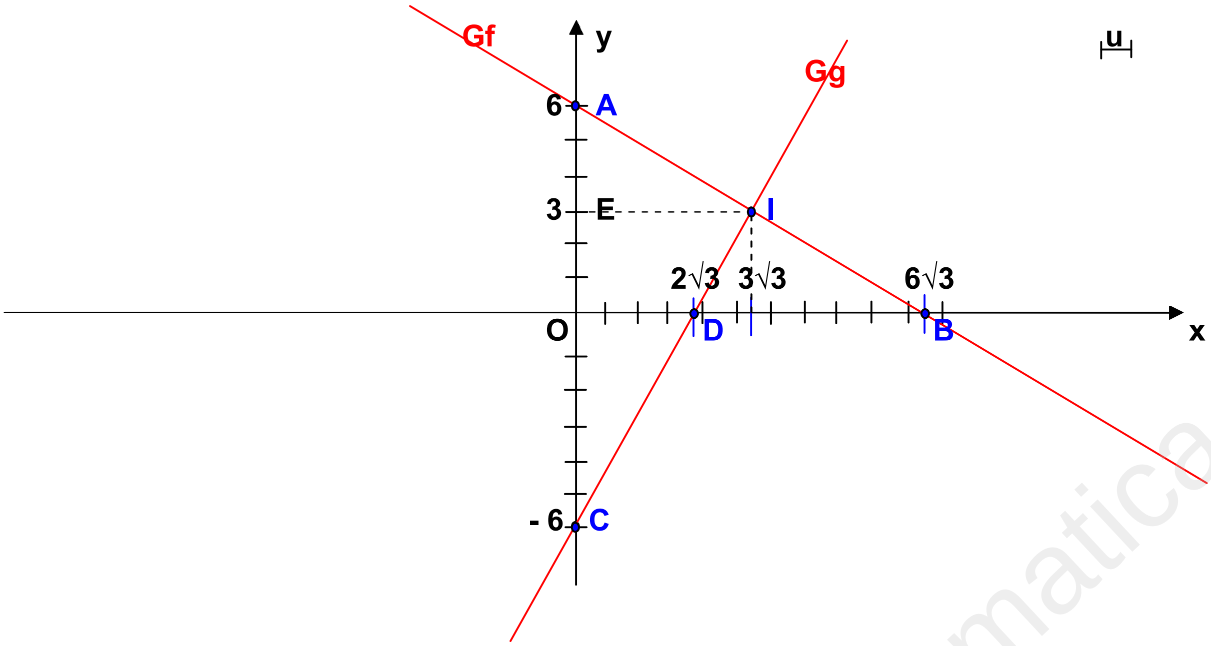
x=0 $\Rightarrow f(0) = \left(-\dfrac{\sqrt{3}}{3}\right)\cdot 0+6=6 \Rightarrow \mathbf{f(0)=6} \Rightarrow \mathbf{A(0; 6)}$

x=0 $\Rightarrow g(0)=\sqrt{3}\cdot 0-6 \Rightarrow \mathbf{g(0)= -6} \Rightarrow \mathbf{C(0; -6)}$

f(x)=0 $\Rightarrow -\dfrac{\sqrt{3}}{3}x + 6 = 0 \Rightarrow \mathbf{x = 6\sqrt{3}} \Rightarrow \mathbf{B(6\sqrt{3}; 0)}$

g(x) = 0 $\Rightarrow \sqrt{3}x-6=0 \Rightarrow \mathbf{x= 2\sqrt{3}} \Rightarrow \mathbf{D(2\sqrt{3}; 0)}$

Reprezint punctele in sistemul de axe xOy si trasez graficul functiei



c) $Gf \perp Gg$ daca $(AB) \perp (CD)$. Verific daca ΔAIC este dreptughic in I

In ΔAEI dr. in E $\Rightarrow AI^2 = AE^2 + EI^2 \Rightarrow AI^2 = 3^2 + (3\sqrt{3})^2 = 9 + 27 = 36 \Rightarrow AI = \sqrt{36} \Rightarrow$ **$AI = 6$ u**

In ΔCEI dr. in E $\Rightarrow CI^2 = CE^2 + EI^2 \Rightarrow CI^2 = 9^2 + (3\sqrt{3})^2 = 81 + 27 = 108 \Rightarrow CI = \sqrt{108} \Rightarrow$ **$CI = 6\sqrt{3}$ u**

In ΔAIC cu **$AI = 6$** ; **$CI = 6\sqrt{3}$** ; **$AC = 12$** se observa ca $AI^2 + CI^2 = AC^2 \Rightarrow$ **ΔAIC dr. in I** $\Rightarrow$

$\Rightarrow m\angle(AIC) = 90° \Rightarrow CI \perp AI \Rightarrow$ **$Gf \perp Gg$**

d) $f(x) = -\dfrac{\sqrt{3}}{3}x + 6 \Rightarrow f(m) = -\dfrac{\sqrt{3}}{3}m + 6$

$g(x) = \sqrt{3}x - 6 \Rightarrow g(m) = \sqrt{3}m - 6$

$3 \cdot f(m) \le g(m) \Leftrightarrow 3 \cdot \left(-\dfrac{\sqrt{3}}{3}m + 6\right) \le \sqrt{3}m - 6 \Rightarrow -\sqrt{3}m + 18 \le \sqrt{3}m - 6 \Rightarrow -\sqrt{3}m - \sqrt{3}m \le -6 - 18 \Rightarrow$

$\Rightarrow -2\sqrt{3}m \le -24 \ / \cdot(-1) \Rightarrow 2\sqrt{3}m \ge 24 \ /:2\sqrt{3} \Rightarrow m \ge 4\sqrt{3} \Rightarrow$ **$m \in [4\sqrt{3}; +\infty)$**