G2. PRISMA TRIUNGHIULARA REGULATA - PROBLEME REZOLVATE

1. Prisma triunghiulara regulata ABCA'B'C' are fața laterala B'BCC' patrat cu diagonala de $6\sqrt{2}$ cm, iar E este proiectia punctului C pe muchia AB. Se cere:

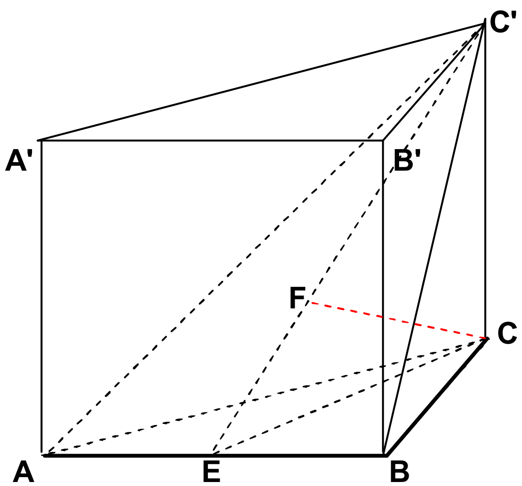
a) Aria laterala, aria totala si volumul prismei.

b) Distanta de la vârful C' la muchia AB.

c) Distanta de la vârful C la planul (C'EB)

d) O functie trigonometrica a unghiului dintre planele (C'EB) si (ABC)

REZOLVARE



a) BC' = BC$\sqrt{2}$ (diagonala patratului) $\Rightarrow 6\sqrt{2} = BC\sqrt{2} \Rightarrow$ **BC = 6 cm** si **CC' = 6 cm**

Aria laterala = Pb·h = 3·6·6 = **54 cm²** ; **Aria totala = Al + 2·Ab ; Ab=** $\frac{l^2\cdot\sqrt{3}}{4}$

Ab = $\frac{6^2\cdot\sqrt{3}}{4} = \frac{36\sqrt{3}}{4} = $ **$9\sqrt{3}$ cm²** $\Rightarrow$ **Aria totala =** $54 + 2\cdot 9\sqrt{3}$ = **($54 + 18\sqrt{3}$) cm²** .

Volumul = Ab·h = $9\sqrt{3}\cdot 6$ = **$54\sqrt{3}$ cm³** .

b) d(C' ; AB)

$$\left.\begin{array}{l} C'C \perp (ABC) \\ CE \perp AB \\ CE, AB \subset (ABC) \end{array}\right\} \Rightarrow C'E \perp AB \Rightarrow d(C' ; AB) = C'E$$

In ΔABC echilateral cu CE=inaltime $\Rightarrow$ **CE** = $\frac{l\sqrt{3}}{2} = \frac{6\sqrt{3}}{2} =$ **$3\sqrt{3}$ cm**

In ΔC'CE dr. $\Rightarrow C'E^2 = C'C^2 + CE^2 \Rightarrow C'E^2 = 6^2 + (3\sqrt{3})^2 = 36 + 27 = 63 \Rightarrow$ **C'E = $\sqrt{63}$ = $3\sqrt{7}$ cm.**

c) d(C; (C'EB))

$$\left.\begin{array}{l} CE \perp EB \\ EC' \perp EB \\ CE, EC' \subset (C'CE) \end{array}\right\} \Rightarrow EB \perp (C'CE)$$

Construiesc $CF \perp EC'$; deoarece $CF \subset (C'CE)$ si $EB \perp (C'CE) \Rightarrow EB \perp CF \Rightarrow CF \perp EB$

$$\text{Din}\quad \left.\begin{array}{l} CF \perp EC' \\ CF \perp EB \\ EC', EB \subset (C'EB) \end{array}\right\} \Rightarrow \mathbf{CF \perp (C'EB) \Rightarrow d(C; (C'EB)) = CF}$$

In $\Delta C'CE$ dr. (cu $CF \perp C'E$) $\Rightarrow CF = \dfrac{CC' \cdot CE}{C'E} = \dfrac{6 \cdot 3\sqrt{3}}{3\sqrt{7}} = \dfrac{6\sqrt{21}}{7} \Rightarrow \mathbf{CF = \dfrac{6\sqrt{21}}{7}\ cm}$.

d) tg $\angle$((C'EB) ; (ABC))

$$\left.\begin{array}{l} (C'EB) \cap (ABC) = EB \\ C'E \perp EB\ ;\ C'E \subset (C'EB) \\ CE \perp EB\ ;\ CE \subset (ABC) \end{array}\right\} \Rightarrow \angle((C'EB) ; (ABC)) = \angle (C'E ; CE) = \angle\mathbf{(C'EC)}$$

In $\Delta C'CE$ dr $\Rightarrow tg\ \angle(C'EC) = \dfrac{C'C}{CE} = \dfrac{6}{3\sqrt{3}} = \dfrac{2\sqrt{3}}{3} \Rightarrow$ $\mathbf{tg\ \angle(C'EC) = \dfrac{2\sqrt{3}}{3}}$.

2. O prisma triunghiulara regulata ABCA'B'C' are apotema bazei de 2√3 cm, iar planul (B'AC) face cu planul (ABC) un unghi cu masura de 60°. Punctul M este mijlocul muchiei AB. Se cere:

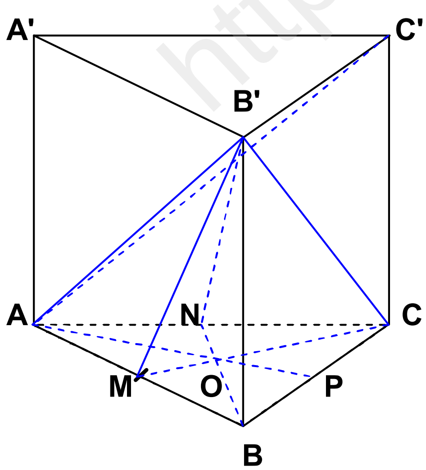
a) Volumul si aria laterala a prismei;

b) Distanta de la punctul B' la dreapta CM;

c) Distanta de la varful C' la planul (ACB')

d) unghiul dintre planele (MCB') si (ABB')

REZOLVARE



a) *Calculez muchia bazei.*

Apotema bazei este $\perp$ din centrul bazei pe o latura a bazei

$OM \perp AB$; $OM = \dfrac{1}{3} \cdot CM \Rightarrow CM = 3 \cdot OM$

Apotema bazei = $OM = 2\sqrt{3}$ cm $\Rightarrow$ inaltimea bazei = $\mathbf{CM = 6\sqrt{3}\ cm}$

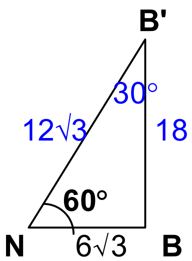
In ΔABC echilat. $\Rightarrow CM = \dfrac{l\sqrt{3}}{2} \Rightarrow 6\sqrt{3} = \dfrac{l\sqrt{3}}{2} \Rightarrow l\sqrt{3} = 12\sqrt{3} \Rightarrow$

$\Rightarrow$ l = 12 cm $\Rightarrow$ **AB = 12 cm**

Identific unghiul dintre planele (B'AC) si (ABC) apoi calculez inaltimea prismei.

$\left.\begin{array}{l}(B'AC) \cap (ABC) = AC \\ B'N \perp AC;\ B'N \subset (B'AC) \\ BN \perp AC\ ;\ BN \subset (ABC)\end{array}\right\} \Rightarrow \angle((B'AC)\ ;\ (ABC)) = \angle(B'N\ ;\ BN) = \angle\ (B'NB) \Rightarrow$ **$m\angle(B'NB) = 60°$**

$\left.\begin{array}{l}B'B \perp (ABC) \\ BN \perp AC \\ BN,\ AC \subset (ABC)\end{array}\right\} \Rightarrow$ **$B'N \perp AC$**



In $\Delta B'BC$ dr. daca $m\angle(B'NB)=60° \Rightarrow m\angle(NB'B) = 30° \Rightarrow NB = \dfrac{NB'}{2} \Rightarrow$

$\Rightarrow 6\sqrt{3} = \dfrac{NB'}{2} \Rightarrow NB' = 12\sqrt{3}$ cm

In $\Delta B'BC$ dr. $\Rightarrow B'B^2 = B'N^2 - BN^2 \Rightarrow B'B^2 = (12\sqrt{3})^2 - (6\sqrt{3})^2 = 432 - 108 = 324 \Rightarrow$

$\Rightarrow B'B = \sqrt{324} \Rightarrow$ **$B'B = 18$ cm**

Aria bazei $= \dfrac{l^2\sqrt{3}}{4} = \dfrac{12^2\cdot\sqrt{3}}{4} = 36\sqrt{3}\ cm^2$

Volumul prismei = Ab·h $= 36\sqrt{3} \cdot 18 =$ **$648\sqrt{3}\ cm^3$**

Aria laterala = Pb·h $= 3 \cdot 12 \cdot 18 =$ **$648\ cm^2$.**

b) d(B' ; CM)

$\left.\begin{array}{l}B'B \perp (ABC) \\ BM \perp CM \\ BM,\ CM \subset (ABC)\end{array}\right\} \Rightarrow B'M \perp CM \Rightarrow$ **d(B' ; CM) = B'M**

In $\Delta B'BM$ dr. in B $\Rightarrow B'M^2 = B'B^2 + BM^2 \Rightarrow B'M^2 = 18^2 + 6^2 = 324 + 36 = 360 \Rightarrow B'M = \sqrt{360} \Rightarrow$

$B'M = 6\sqrt{10}$ cm

c) d(C' ; (ACB'))

Formez intre punctul C' si planul (ACB') piramida C'ACB' si scriu volumul ei in 2 moduri

Construiesc $AP \perp BC$, deoarece $(ABC) \perp (B'BCC')$ si $AP \subset (ABC) \Rightarrow AP \perp CC'$

Daca $AP \perp BC$ si $AP \perp CC' \Rightarrow AP \perp (B'BCC') \Rightarrow d(A\ ;\ (B'CC') = AP$

$V_{piramidei\ C'ACB'} = \dfrac{\text{Aria } \Delta B'CC' \cdot AP}{3} = \dfrac{\text{Aria } \Delta ACB' \cdot d(C'\ ;\ (ACB'))}{3} \Rightarrow d(C';(ACB')) = \dfrac{\text{Aria } \Delta B'CC' \cdot AP}{\text{Aria } \Delta ACB'}$

Aria $\Delta B'CC' = \dfrac{\text{Aria } B'BCC'}{2} = \dfrac{12 \cdot 18}{2} = 108\ cm^2$; Aria $\Delta ACB' = \dfrac{AC \cdot B'N}{2} = \dfrac{12 \cdot 12\sqrt{3}}{2} = 72\sqrt{3}\ cm^2$

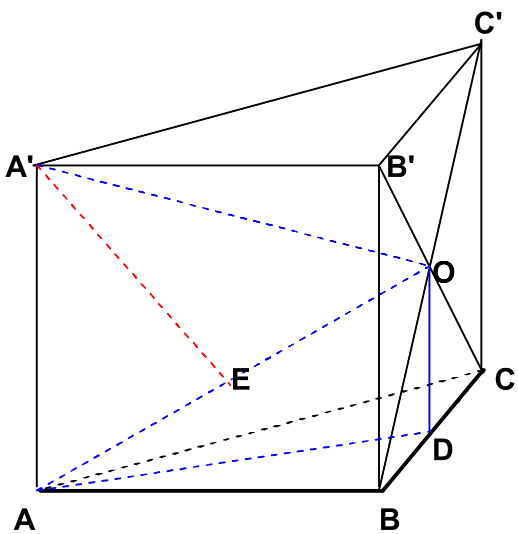
$d(C';(ACB')) = \dfrac{108 \cdot 6\sqrt{3}}{72\sqrt{3}} = \dfrac{108}{12} = 9cm$

d) m∠((MCB') ; (ABB'))

$\left.\begin{array}{l} CM \perp B'M \text{ (demonstrat la punctul \textbf{b}))} \\ CM \subset (MCB') \\ B'M \subset (ABB') \end{array}\right\} \Rightarrow (MCB') \perp (ABB') \Rightarrow m\angle((MCB');(ABB')) = 90^\circ.$

3) O prisma triunghiulara regulata ABCA'B'C' are aria totala egala cu $(192 + 32\sqrt{3})$ cm^2 , aria bazei egala cu $16\sqrt{3}$ cm^2, iar punctul O este centrul feței B'BCC'. Se cere:
a) Volumul prismei ; b) Unghiul dintre segmentul AO si planul (ABC) ; c) Distanta de la varful A' la segmentul AO.

REZOLVARE



a) Aria totala = Aria laterala + 2·Aria bazei ⇒

$192 + 32\sqrt{3}$ = Aria laterala + $2 \cdot 16\sqrt{3}$ ⇒

Aria laterala = 192 cm^2

Aria bazei = $\dfrac{l^2\cdot\sqrt{3}}{4} \Rightarrow 16\sqrt{3} = \dfrac{l^2\cdot\sqrt{3}}{4} \Rightarrow l^2\sqrt{3} = 64\sqrt{3} \Rightarrow$

$\Rightarrow l^2 = 64 \Rightarrow l = \sqrt{64} \Rightarrow$ **l = 8 cm**

Aria laterala = Pb·h ⇒ 192 = 3·8·h ⇒ **h = 8cm**

Volumul = Ab · h = $16\sqrt{3} \cdot 8 = 128\sqrt{3}$ cm^3 .

b) m ∠ (AO ; (ABC))

$\left.\begin{array}{l} OD \perp (ABC) \\ A \in (ABC) \end{array}\right\} \Rightarrow$ AD este proiectia lui AO pe planul (ABC) ⇒ ∠(AO;(ABC)) = ∠(AO;AD) = ∠(OAD)

In ΔABC echilateral cu AD inaltime ⇒ AD = $\dfrac{l\sqrt{3}}{2} = \dfrac{8\sqrt{3}}{2} = 4\sqrt{3}$ cm

Deoarece O este centrul fetei B'BCC' ⇒ OD = $\dfrac{BB'}{2} = \dfrac{8}{2} = 4$ cm

In ΔODA dr. in D ⇒ $AO^2 = AD^2 + DO^2 \Rightarrow AO^2 = (4\sqrt{3})^2 + 4^2 = 48 + 16 = 64 \Rightarrow AO = \sqrt{64} \Rightarrow$ AO = 8 cm

In ΔODA dr. in D , se observa ca OD = $\dfrac{AO}{2}$ ⇒ m∠(OAD) = 30°

c) d(A' ; AO)

Formez intre punctul **A'** si segmentul **AO** triunghiul **A'AO**

In ΔA'AO construiesc A'E ⊥ AO ⇒ **d(A' ; AO) = A'E**

Deoarece AO = A'O = A'A = 8cm ⇒ ΔA'AO este echilateral, cu A'E inaltime ⇒ A'E = $\dfrac{l\sqrt{3}}{2} = \dfrac{8\sqrt{3}}{2} \Rightarrow$

⇒ A'E = $4\sqrt{3}$ cm ⇒ **d(A' ; AO) = $4\sqrt{3}$ cm.**