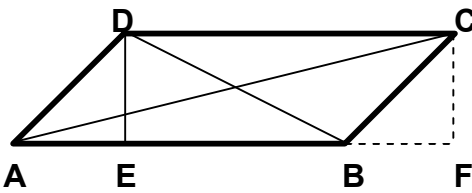


### C. Paralelogramul, rombul, dreptunghiul, patratul

- 1) Un paralelogram are aria =  $40 \text{ cm}^2$ , latura mare  $10 \text{ cm}$  si unghiul obtuz de  $135^\circ$ . Se cere:  
**a)** Perimetrul paralelogramului ; **b)** diagonalele paralelogramului

#### REZOLVARE



Construim  $DE \perp AB \Rightarrow \triangle DEA$   
 Daca  $m(\angle ADC) = 135^\circ \Rightarrow m(\angle DAE) = 45^\circ$   
 $\Rightarrow \triangle DEA$  este dreptunghic isoscel  $\Rightarrow AE = DE$   
 Aria  $ABCD = AB \cdot DE \Rightarrow 10 \cdot DE = 40 \Rightarrow DE = 4 \text{ cm} \Rightarrow AE = 4 \text{ cm}$

In  $\triangle ADE \Rightarrow AD^2 = AE^2 + DE^2 \Rightarrow AD^2 = 16 + 16 = 32 \Rightarrow AD = 4\sqrt{2} \text{ cm}$  Deci  **$AB = DC = 10 \text{ cm}$**  ;  **$AD = BC = 4\sqrt{2} \text{ cm}$**

**a) Perimetrul ABCD =  $2(AB + AD) = 2(10 + 4\sqrt{2}) = 4(5 + 2\sqrt{2}) \text{ cm}$**

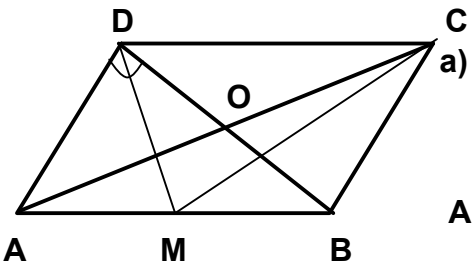
**b) Diagonala DB se calculeaza din  $\triangle DEB$ , unde  $DE = 4 \text{ cm}$  si  $EB = 10 - 4 = 6 \text{ cm}$**   
 In  $\triangle DEB \Rightarrow DB^2 = DE^2 + EB^2 \Rightarrow DB^2 = 16 + 36 = 52 \Rightarrow DB = 2\sqrt{13} \text{ cm}$

Pentru a calcula diagonala CA se construiesc  $CF \perp AB \Rightarrow \triangle CFA$  unde  $AF = 10 + 4 = 14 \text{ cm}$  ( $BF = AE = 4 \text{ cm}$ )  
 In  $\triangle CFA \Rightarrow CA^2 = CF^2 + AF^2 \Rightarrow CA^2 = 16 + 196 = 212 \Rightarrow CA = 2\sqrt{53} \text{ cm}$ .

- 2) Fie ABCD un paralelogram in care  $AD \perp BD$ ,  $AB = 2 \cdot BC$ ,  $AB = 12 \text{ cm}$ . Se cere:

- a)** Perimetrul si aria paralelogramului ; **b)** Perimetrul triunghiului ABC  
**c)** Aria triunghiului DMC, unde M este mijlocul laturii AB

#### REZOLVARE



**a)**  $AD \perp BD \Rightarrow \triangle ADB$  dreptunghic; daca  $AB = 2 \cdot BC \Rightarrow AB = 2 \cdot AD$   
 $AB = 12 \text{ cm} \Rightarrow AD = 6 \text{ cm} \Rightarrow$  Perimetrul ABCD =  **$36 \text{ cm}$**

In  $\triangle ADB \Rightarrow DB^2 = AB^2 - AD^2 = 144 - 36 = 108 \Rightarrow DB = 6\sqrt{3} \text{ cm} \Rightarrow$  Aria ABCD =  **$DB \cdot AD = 36\sqrt{3} \text{ cm}^2$** .

**b)** In  $\triangle ADO$  dreptunghic ;  $DO = DB/2 = 3\sqrt{3} \text{ cm}$  ;  $AD = 6 \text{ cm} \Rightarrow AO^2 = AD^2 + DO^2 = 36 + 27 = 63 \Rightarrow AO = 3\sqrt{7} \text{ cm} \Rightarrow AC = 2 \cdot AO = 6\sqrt{7} \text{ cm} \Rightarrow$  Perimetrul  $\triangle ABC = 12 + 6 + 6\sqrt{7} =$   **$(18 + 6\sqrt{7}) \text{ cm}$**

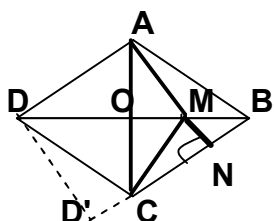
**c)** In  $\triangle ADB$  ( $m\angle D = 90^\circ$ ) ; DM- mediana  $\Rightarrow DM = AB/2 \Rightarrow DM = 6 \text{ cm}$

$DM = AD = 6 \text{ cm}$  ;  $DC = AB = 12 \text{ cm}$  ;  $CM = DB$  ( $\triangle DMB \cong \triangle CMB$ )  $\Rightarrow \triangle DMC \cong \triangle DAB \Rightarrow$  Aria  $\triangle DMC =$   $\frac{\text{Aria ABCD}}{2}$   
 $\Rightarrow$  **Aria  $\triangle DMC = 18\sqrt{3} \text{ cm}^2$ .**

- 3) Fie ABCD un romb in care  $AB = 5 \text{ cm}$ ,  $AC = 6 \text{ cm}$ , iar  $M \in BD$  astfel incit  $\frac{MD}{BD} = \frac{3}{4}$ . Se cere:

- a)** Aria rombului ; **b)** Perimetrul  $\triangle ACM$  ; **c)** distanta de la M la BC .

#### REZOLVARE



In  $\triangle AOB$  ( $m\angle O = 90^\circ$ )  $AB = 5 \text{ cm}$  ;  $AO = AC/2 = 3 \text{ cm} \Rightarrow OB^2 = AB^2 - AO^2 \Rightarrow OB^2 = 25 - 9 = 16 \Rightarrow OB = 4 \text{ cm} \Rightarrow DB = 8 \text{ cm}$

**a) Aria ABCD =  $\frac{AC \cdot DB}{2} = \frac{6 \cdot 8}{2} = 24 \text{ cm}^2$ .**

$$\text{b) } \frac{MD}{BD} = \frac{3}{4} \Rightarrow MD = \frac{3}{4} BD \Rightarrow MB = \frac{1}{4} \cdot BD = \frac{1}{4} \cdot 8 = 2 \text{ cm} \Rightarrow \mathbf{OM=MB=2 \text{ cm}}$$

$$\text{In } \Delta AOM (\angle O=90^\circ) \Rightarrow AM^2 = AO^2 + OM^2 \Rightarrow AM^2 = 9 + 4 = 13 \Rightarrow \mathbf{AM=MC=\sqrt{13} \text{ cm}}$$

$$\text{Perimetrul } \Delta AMC = AC + 2 \cdot AM = (6 + 2\sqrt{13}) \text{ cm.}$$

$$\text{c) } MN \perp BC \Rightarrow d(M; BC) = MN ; DD' \perp BC \Rightarrow DD' \parallel MN \Rightarrow \Delta DD'B \sim \Delta MNB \Rightarrow \frac{MN}{DD'} = \frac{MB}{DB} \Rightarrow MN = \frac{DD' \cdot MB}{DB}$$

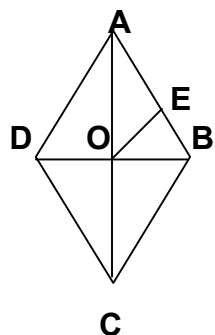
Utilizand aria rombului calculez inaltimea lui si implicit segmentul  $DD'$  (care este inaltime)

$$\text{Aria} = \text{latura} \cdot \text{inaltimea} \Rightarrow \text{inaltimea} = \frac{\text{Aria}}{\text{latura}} \Rightarrow DD' = \frac{\text{Aria}_{ABCD}}{AB} = \frac{24}{5} \text{ cm}$$

$$\mathbf{MN} = \frac{24}{5} \cdot 2 \cdot \frac{1}{8} = \frac{6}{5} \text{ cm.}$$

4) Raportul diagonalelor unui romb este  $3/4$ , iar perimetrul rombului este  $40 \text{ cm}$ . Se cere: a) aria rombului ; b) inaltimea rombului ; c) distanta din centrul rombului la o latura a sa.

#### REZOLVARE



$$\frac{DB}{AC} = \frac{3}{4} \Rightarrow DB = 3k \text{ si } AC = 4k ; \text{ Daca perimetrul este } 40 \Rightarrow AB = 10 \text{ cm}$$

$$\text{In } \Delta AOB \Rightarrow AO^2 + OB^2 = AB^2 \Rightarrow \left(\frac{4k}{2}\right)^2 + \left(\frac{3k}{2}\right)^2 = 100 \Rightarrow \frac{25k^2}{4} = 100 \Rightarrow k = 4$$

$$\Rightarrow \mathbf{DB=12 \text{ cm} ; AC=16 \text{ cm}}$$

$$\mathbf{a) \text{ Aria } ABCD = \frac{DB \cdot AC}{2} = \frac{12 \cdot 16}{2} = 96 \text{ cm}^2}$$

$$\text{b) Aria romb} = \text{inaltimea} \cdot \text{latura} \Rightarrow \text{inaltimea} = \text{Aria} / \text{latura} \Rightarrow \mathbf{\text{inaltimea romb} = 96/10 = 9,6 \text{ cm}}$$

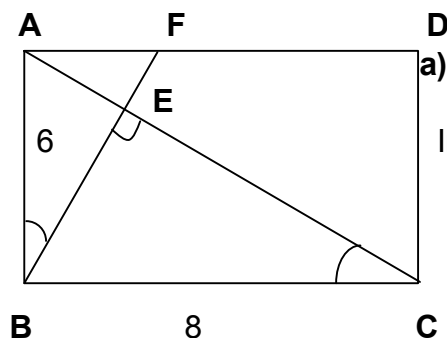
$$\text{c) } OE \perp AB \Rightarrow d(O; AB) = OE . \text{ In } \Delta AOB (\angle O=90^\circ) \Rightarrow OE = \frac{OA \cdot OB}{AB} = \frac{6 \cdot 8}{10} = \frac{24}{5} \Rightarrow \mathbf{OE = \frac{24}{5} \text{ cm.}}$$

5) Fie dreptunghiul ABCD cu  $AB=6 \text{ cm}$  ;  $BC=8 \text{ cm}$ . Se cere:

a) Distanța de la virful B la diagonala AC.

b) Daca E este proiectia punctului B pe AC iar F este intersectia lui BE cu AD ( $F \in AD$ ), aflati [AF]

#### REZOLVARE



$$\text{a) } BE \perp AC \Rightarrow d(B; AC) = BE . \text{ In } \Delta ABC (\angle B=90^\circ) \Rightarrow BE = \frac{AB \cdot BC}{AC}$$

$$\text{In } \Delta ABC (\angle B=90^\circ) \Rightarrow AC^2 = AB^2 + BC^2 \Rightarrow AC^2 = 36 + 64 = 100$$

$$\Rightarrow AC = 10 \text{ cm} \Rightarrow BE = \frac{6 \cdot 8}{10} = \frac{24}{5} \Rightarrow \mathbf{BE = \frac{24}{5} \text{ cm}}$$

b)  $BE \perp AC \Rightarrow E$  este proiectia punctului B pe AC

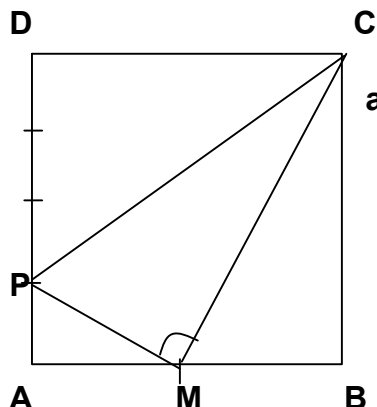
$$m(\angle ABE) + m(\angle EBC) = 90^\circ$$

$$m(\angle ECB) + m(\angle EBC) = 90^\circ \Rightarrow m(\angle ABE) = m(\angle ECB) \Rightarrow \triangle BAF \sim \triangle ABC \Rightarrow \frac{BF}{AC} = \frac{AF}{AB} = \frac{AB}{BC} \Rightarrow$$

$$\frac{6}{6} = \frac{6}{8} \Rightarrow AF = \frac{6 \cdot 6}{8} \Rightarrow AF = \frac{9}{2} \text{ cm.}$$

6) In dreptunghiul ABCD se considera M, mijlocul laturii AB si  $P \in (AD)$ , astfel incit  $DP = 3 \cdot AP$ .  
Sa se arate ca daca  $MP \perp MC$ , atunci ABCD este patrat

### REZOLVARE



a) Notam  $AB = a$  si  $AD = b$ ; Daca  $DP = 3 \cdot AP \Rightarrow DP = \frac{3b}{4}$  si  $AP = \frac{b}{4}$

Daca M este la mijlocul lui AB  $\Rightarrow AM = MB = \frac{a}{2}$

Din  $\triangle PAM$  dr.  $\Rightarrow PM^2 = AM^2 + PA^2 = \frac{a^2}{4} + \frac{b^2}{16} = \frac{4a^2 + b^2}{16}$  (1)

Din  $\triangle CMB$  dr.  $\Rightarrow CM^2 = MB^2 + CB^2 \Rightarrow CM^2 = \frac{a^2}{4} + b^2 = \frac{a^2 + 4b^2}{4}$  (2)

Din  $\triangle PDC$  dr.  $\Rightarrow PC^2 = DP^2 + DC^2 \Rightarrow PC^2 = \frac{9b^2}{16} + a^2 = \frac{9b^2 + 16a^2}{16}$  (3)

Din relatiile (1), (2), (3) aplicind teorema lui Pitagora in  $\triangle PMC \Rightarrow a^2 = b^2 \Rightarrow a = b \Rightarrow ABCD$  patrat