EXERCITII SI PROBLEME DE ALGEBRA REZOLVATE

A. OPERATII CU NUMERE REALE

a)

1) $-(-2)+(-5)-|-5|+|-6|-8+9 = +2-5-5+6-8+9 = +17-18 = \mathbf{-1}$

2) $(-3)\cdot(-5)-(+8):(-4)-(-4)\cdot(-2) = +15-(-2)-(+8) = 15+2-8 = 17-8 = \mathbf{9}$

3) $6-6:2\cdot3+3\cdot6:18 = 6-3\cdot3+18:18 = 6-9+1 = 7-9 = \mathbf{-2}$

4) $8+8:6\cdot9-9:6:2\cdot4 = 8+\frac{8^{(2}}{6}\cdot9-\frac{9^{(3}}{6}:2\cdot4 = 8+\frac{4}{3}\cdot\frac{9^{3}}{1}-\frac{3}{2}\cdot\frac{1}{2}\cdot\frac{4}{1} = 8+12-3 = \mathbf{17}$

5) $[3^{28}:3^{18}-(2^{10})^2+6\cdot6^9]:(2^{10}\cdot3^{10}-8^{20}:4^{20}+3^4:3^{-6}) =$

$= (3^{28-18}-2^{10\cdot2}+6^{1+9}):[(2\cdot3)^{10}-(8:4)^{20}+3^{4-(-6)}] = (3^{10}-2^{20}+6^{10}):(6^{10}-2^{20}+3^{10}) = \mathbf{1}$

6) $2^{300}\cdot3^{200}-2^{100}\cdot6^{200} = (2^3)^{100}\cdot(3^2)^{100}-2^{100}\cdot(6^2)^{100} = 8^{100}\cdot9^{100}-2^{100}\cdot36^{100} = 72^{100}-72^{100} = \mathbf{0}$

7) $-2^2-(-2)^2+(-2)^3-1^{1900}-(-1)^{2006}+2005^0 = -4-(+4)+(-8)-1-(+1)+1 = -4-4-8-1-1+1 = \mathbf{-17}$

8) $1,25\cdot2,(6)-3\frac{1}{3} = \frac{5}{4}\cdot\frac{8^{2}}{3}-\frac{10}{3} = \frac{10}{3}-\frac{10}{3} = \mathbf{0}$

$1,25 = \frac{125^{(25}}{100} = \frac{5}{4}$; $2,(6) = \frac{26-2}{9} = \frac{24^{(3}}{9} = \frac{8}{3} = \frac{1}{3}$; $3\frac{1}{3} = \frac{3\cdot3+1}{3} = \frac{10}{3}$

9) $215,(6)-215,1(6) =$ 215,66666666..... − 215,16666666..... = 0,50000000..... $= \mathbf{0,5}$

10) $\frac{3}{4}+\frac{3}{4}:0,1^{-1}:(\frac{37}{40}-1) = \frac{3}{4}+\frac{3}{4}:(\frac{1}{10})^{-1}:(\frac{37-40}{40}) = \frac{3}{4}+\frac{3}{4}:\frac{10}{1}:\frac{(-3)}{40} =$

$= \frac{3}{4}-\frac{3}{4}\cdot\frac{1}{10}\cdot\frac{40}{3} = \frac{3}{4}-\frac{4)1}{1} = \frac{3-4}{4} = \mathbf{-\frac{1}{4}}$

http://e-profu.ro/matematica

b) Sume

$$1 + 2 + 3 + \ldots\ldots\ldots\ldots\ldots + n = S_n = \frac{n(n+1)}{2}$$

Exemple:

1. $1 + 2 + 3 + \ldots\ldots\ldots\ldots\ldots + 100 = \frac{100 \cdot 101}{2} = 5050$

2. $2 + 4 + 6 + \ldots\ldots + 100 = 2 \cdot (1 + 2 + 3 + \ldots\ldots + 50) = 2 \cdot \frac{50 \cdot 51}{2} = 2550$

3. $10 + 11 + 12 + \ldots\ldots\ldots\ldots\ldots + 50 = S_{50} - S_9 = \frac{50 \cdot 51}{2} - \frac{9 \cdot 10}{2} = 1230$

4. $1 + 3 + 5 + 7 + \ldots + 99 = S_{99} - (2+4+6+\ldots\ldots+98) = S_{99} - 2 \cdot S_{49} = 99 \cdot 50 - 49 \cdot 50 = 50 \cdot 50 = 2500$

5. $\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \ldots\ldots\ldots + \frac{1}{49 \cdot 50} = \frac{3-2}{2 \cdot 3} + \frac{4-3}{3 \cdot 4} + \frac{5-4}{4 \cdot 5} + \ldots\ldots\ldots + \frac{50-49}{49 \cdot 50}$

$$= \frac{3}{2 \cdot 3} - \frac{2}{2 \cdot 3} + \frac{4}{3 \cdot 4} - \frac{3}{3 \cdot 4} + \frac{5}{4 \cdot 5} - \frac{4}{4 \cdot 5} + \ldots\ldots\ldots\ldots + \frac{50}{49 \cdot 50} - \frac{49}{49 \cdot 50} =$$

$$= \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \ldots\ldots + \frac{1}{49} - \frac{1}{50} = \frac{1}{2} - \frac{1}{50} = \frac{24}{50} = \frac{12}{25}$$

6. $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots\ldots\ldots + \frac{1}{90} = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \ldots\ldots\ldots\ldots + \frac{1}{9 \cdot 10}$

mai departe se rezolva ca exercitiul nr.5

7. $\left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{1}{3}\right) \cdot \left(1 - \frac{1}{4}\right) \cdot \ldots\ldots \cdot \left(1 - \frac{1}{10}\right) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \ldots\ldots \cdot \frac{9}{10} = \frac{1}{10}$

c) Ultima cifra a unui numar

Un numar care se termina cu cifra 1 ridicat la orice putere are ultima cifra 1 $(\ldots1)^n = \ldots\ldots1$

Un numar care se termina cu cifra 5 ridicat la orice putere are ultima cifra 5 $(\ldots5)^n = \ldots\ldots5$

Un numar care se termina cu cifra 6 ridicat la orice putere are ultima cifra 6 $(\ldots6)^n = \ldots\ldots6$

Un numar care se termina cu cifra 4 : ridicat la putere impara are ultima cifra 4 $4^1 = 4$

ridicat la putere para are ultima cifra 6 $4^2 = 16$

Un numar care se termina cu cifra 9 : ridicat la putere impara are ultima cifra 9 $9^1 = 9$

ridicat la putere para are ultima cifra 1 $9^2 = 81$

http://eprofu.ro/matematica

Exercitii:

a) $2^{425} = 2 \cdot 2^{424} = 2 \cdot 2^{4 \cdot 106} = 2 \cdot (2^4)^{106} = 2 \cdot 16^{106} = 2 \cdot (.....6) = ...2$, ultima cifra este **2**

b) $3^{479} = 3 \cdot 3^{478} = 3 \cdot 3^{2 \cdot 239} = 3 \cdot (3^2)^{239} = 3 \cdot 9^{239} = 3 \cdot (.....9) = ...7$, ultima cifra este **7**

c) Aflarea a *n*-a zecimala a unui numar zecimal periodic

Exercitii:
1) Aflati a 101 zecimala a numarului 2,34(125) .

Se rezolva parcurgand urmatoarele etape:

* 2,34(125) = 2,34125125125125.....125.....

* **din 101 se scade numarul de cifre care sunt in afara perioadei** : 101 - 2 = 99

* **rezultatul care ramine se imparte la numarul de cifre din perioada** : 99 :3 = 33 rest 0

* daca restul este: **0** atunci zecimala 101 este **ultima cifra din perioada**, adica 5
  1 atunci zecimala 101 este **prima cifra din perioada**, adica 1
  2 atunci zecimala 101 este **a doua cifra din perioada**, adica 2

2) Aflati a 2001 zecimala a numarului 0,134(32156)

2001 - 3 = 1998 1998 : 5 = 399 **rest 3** , zecimala 2001 este **a treia cifra din perioada**, adica **1**

3) Aflati a 205 zecimala a numarului 53 / 7

53 / 7 = 7,57142857142857= 7,57(142857)

205 - 2 = 203 ; 203 : 6 = 33 **rest 5** ; zecimala 205 este **a cincea cifra din perioada** , adica **5**

http://eprofu.ro/matematica