O. EXPRESII ALGEBRICE - PROBLEME REZOLVATE

1) Fie expresia: $E(x) = \left(\frac{x}{x^2 - 9} + \frac{2}{3 + x} + \frac{1}{3 - x} \right) : \frac{2x - 9}{x^2 - 2x - 15}$

a) Determinati valorile lui *x* pentru care *E(x)* are sens

b) Determinati $a \in Z$ pentru care $E(a) \in Z$

c) Rezolvati in N inecuatia $(x + 3) \cdot E(x) \leq 0$

REZOLVARE

a) Egalez numitorii fractiilor cu 0, iar la fractia care este dupa semnul : egalez si numaratorul cu 0

$x^2 - 9 = 0 \Rightarrow (x - 3)(x + 3) = 0 \Rightarrow x - 3 = 0 \Rightarrow \mathbf{x = 3}$

$x + 3 = 0 \Rightarrow \mathbf{x = -3}$

$x^2 - 2x - 15 = 0 \Rightarrow x^2 - 5x + 3x - 15 = 0 \Rightarrow x(x-5) + 3(x-5) = 0 \Rightarrow (x - 5)(x + 3) = 0 \Rightarrow x - 5 = 0 \Rightarrow \mathbf{x = 5}$

$x + 3 = 0 \Rightarrow \mathbf{x = -3}$

$2x - 9 = 0 \Rightarrow 2x = 9 \Rightarrow \mathbf{x = \frac{9}{2}}$

Deoarece E(x) are sens $\Rightarrow$ $\mathbf{x \in R - \{ -3; 3; \frac{9}{2} ; 5 \}}$

b) Mai intai aduc *E(x)* la foma cea mai simpla apoi determin *E(a),* inlocuind in *E(x)* pe *x* cu *a*

$$E(x) = \left[\frac{x}{(x - 3)(x + 3)} + \frac{2^{(x-3)}}{x + 3} - \frac{1^{(x+3)}}{x - 3} \right] : \frac{2x - 9}{(x - 5)(x + 3)} = \frac{x + 2x - 6 - x - 3}{(x - 3)(x + 3)} \cdot \frac{(x - 5)(x + 3)}{2x - 9} =$$

$$= \frac{2x - 9}{(x - 3)(x + 3)} \cdot \frac{(x - 5)(x + 3)}{2x - 9} = \frac{x - 5}{x + 3} \Rightarrow \mathbf{E(x) = \frac{x - 5}{x + 3}} \Rightarrow \mathbf{E(a) = \frac{a - 5}{a + 3}}$$

$E(a) \in Z$ daca $\mathbf{a + 3 \mid a + 3}$

$\mathbf{a + 3 \mid a - 5}$ /·(-1) $\Rightarrow$ $a + 3 \mid -a + 5$ (+)

$a + 3 \mid 8 \Rightarrow \mathbf{a + 3 \in D_8} \Rightarrow$

a+3=1	a+3= -1	a+3=2	a+3= -2	a+3=4	a+3= -4	a+3=8	a+3= -8
a = -2	a = -4	a = -1	a = -5	a = 1	a= -7	a = 5	a = -11

Din conditia de existenta a fractiilor $\Rightarrow$ $a \neq 5 \Rightarrow \mathbf{a \in \{ -11, -7, -5, -4, -2, -1, 1\}}$

c) $\mathbf{(x + 3) \cdot \frac{x - 5}{x + 3} \leq 0} \Rightarrow x - 5 \leq 0 \Rightarrow x \leq 5 \Rightarrow x \in (-\infty ; 5]$

Deoarece $x \in N \Rightarrow x \in \{0, 1, 2, 3, 4, 5\}$

Din conditiile de existenta a fractiilor $\Rightarrow x \neq \{3, 5\}$

$\Rightarrow \mathbf{x \in \{0, 1, 2, 4\}}$

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2) Fie expresia $E(x) = \frac{1}{2} \cdot \left[\left(\frac{1}{3-x} \cdot \frac{x^2-9}{x^2-x+1} + \frac{1}{x-3} \right) : \frac{10-x}{x^3+1} - 3 \right]$

a) Determinati valorile lui *x* in care *E(x)* nu este definita.

b) Verificati daca $E(a) = \frac{5-a}{a-3}$

c) Determinati $a \in Z$**, astfel incat** $E(a) \in N$

d) Determinati elementele multimii $A = \{ x \in N^* \mid E(x) \leq -1 \}$

REZOLVARE

a) $3 - x = 0 \Rightarrow -x = -3/\cdot(-1) \Rightarrow \mathbf{x = 3}$

$x^3 + 1 = 0$; Descompun $x^3 + 1$ utilizand formula de calcul prescurtat $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$x^3 + 1 = (x + 1)(x^2 - x + 1)$; $\Rightarrow (x + 1)(x^2 - x + 1) = 0 \Rightarrow x + 1 = 0 \Rightarrow \mathbf{x = -1}$

$10 - x = 0 \Rightarrow -x = -10/\cdot(-1) \Rightarrow \mathbf{x = 10}$

E(x) nu este definita pentru $x \in \{-1, 3, 10\}$

b) $E(x) = \frac{1}{2} \cdot \left[\left(\frac{-1}{x-3} \cdot \frac{(x-3)(x+3)}{x^2-x+1} + \frac{1}{x-3} \right) : \frac{10-x}{(x+1)(x^2-x+1)} - 3 \right]$

$E(x) = \frac{1}{2} \cdot \left[\left(\frac{x-3)}{x-3} \cdot \frac{-x-3}{x^2-x+1} + \frac{x^2-x+1)}{x-3} \right) \cdot \frac{(x+1)(x^2-x+1)}{10-x} - 3 \right]$

$E(x) = \frac{1}{2} \cdot \left(\frac{-x^2-3x+3x+9+x^2-x+1}{(x-3)(x^2-x+1)} \cdot \frac{(x+1)(x^2-x+1)}{10-x} - 3 \right) = \frac{1}{2} \cdot \left(\frac{10-x}{x-3} \cdot \frac{x+1}{10-x} - 3 \right) =$

$= \frac{1}{2} \cdot \left(\frac{x+1}{x-3} - \frac{x-3)}{1} 3 \right) = \frac{1}{2} \cdot \frac{x+1-3x+9}{x-3} = \frac{1}{2} \cdot \frac{-2x+10}{x-3} = \frac{1}{2} \cdot \frac{2(5-x)}{x-3} = \frac{5-x}{x-3}$

$E(x) = \frac{5-x}{x-3} \Rightarrow \mathbf{E(a) = \frac{5-a}{a-3}}$

c) $\mathbf{E(a) \in N \Rightarrow a - 3 \mid a - 3}$

$\mathbf{a - 3 \mid 5 - a \Rightarrow a - 3 \mid (a - 3) + (5 - a) \Rightarrow a - 3 \mid 2 \Rightarrow a - 3 = D_2(+)}$

$a - 3 = 1 \Rightarrow \mathbf{a = 4}$; $a - 3 = 2 \Rightarrow \mathbf{a = 5} \Rightarrow \mathbf{a \in \{4, 5\}}$

d) $E(x) \leq -1 \Rightarrow \frac{5-x}{x-3} \leq -1 \Rightarrow \frac{5-x}{x-3} + 1 \leq 0 \Rightarrow \frac{5-x+x-3}{x-3} \leq 0 \Rightarrow \frac{2}{x-3} \leq 0$

Fractia este negativa daca numaratorul si numitorul au semne opuse.

Deoarece $2 > 0$, fractia va fi negativa pentru $x - 3 < 0 \Rightarrow x < 3 \Rightarrow x \in (-\infty ; 3)$

Deoarece $x \in N^* \Rightarrow \mathbf{A = \{1, 2\}}$

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3) Fie expresia $E(x) = \frac{2x}{x-1} + \frac{4}{2x^2 - x} \cdot \left[\frac{2-x}{1-4x^2} \cdot \frac{1-16x^3:(-4x)+4x}{2x^2 + x} - \frac{x+2}{2x^2 - 3x^2:3x} \right]^{-1}$

a) Determinati valorile lui $x \in R$, pentru care E(x) are sens.

b) Verificati daca $E(x) = \frac{x+1}{x-1}$

c) Determinati $a \in Z$ astfel incat $E(a) \in Z$

REZOLVARE

a) $x - 1 = 0 \Rightarrow x = 1$; $2x^2 - x = 0 \Rightarrow x(2x - 1) = 0 \Rightarrow x = 0$

$2x - 1 = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

$1 - 4x^2 = 0 \Rightarrow (1 - 2x)(1 + 2x) = 0 \Rightarrow 1 - 2x = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

$\Rightarrow 1 + 2x = 0 \Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}$

E(x) are sens daca $x \in R \setminus \{ -\frac{1}{2}, 0, \frac{1}{2}, 1 \}$

b) $E(x) = \frac{2x}{x-1} + \frac{4}{x(2x-1)} \cdot \left[\frac{2-x}{(1-2x)(1+2x)} \cdot \frac{1+4x^2+4x}{x(2x+1)} - \frac{x+2}{2x^2-x} \right]^{-1}$

$E(x) = \frac{2x}{x-1} + \frac{4}{x(2x-1)} \cdot \left[\frac{2-x}{(1-2x)(1+2x)} \cdot \frac{(1+2x)^2}{x(2x+1)} - \frac{x+2}{x(2x-1)} \right]^{-1}$

$E(x) = \frac{2x}{x-1} + \frac{4}{x(2x-1)} \cdot \left[\frac{x-2}{x(2x-1)} - \frac{x+2}{x(2x-1)} \right]^{-1} = \frac{2x}{x-1} + \frac{4}{x(2x-1)} \cdot \left[\frac{-4}{x(2x-1)} \right]^{-1}$

$E(x) = \frac{2x}{x-1} + \frac{4}{x(2x-1)} \cdot \frac{x(2x-1)}{(-4)} = \frac{2x}{x-1} - 1 = \frac{2x-x+1}{x-1} = \frac{x+1}{x-1} \Rightarrow \mathbf{E(x) = \frac{x+1}{x-1}}$

c) $E(a) = \frac{a+1}{a-1}$; $E(a) \in Z$ daca $a - 1 \mid a - 1$

$a - 1 \mid a + 1 \Rightarrow a - 1 \mid (a+1) - (a-1) \Rightarrow a - 1 \mid 2 \Rightarrow a - 1 = D2$

$a - 1 = 1$ $a - 1 = -1$ $a - 1 = 2$ $a - 1 = -2$

$a = 2$ **$a = 0$** **$a = 3$** **$a = -1$**

Din conditiile de existenta a fractiilor $\Rightarrow x \neq 0 \Rightarrow a \neq 0 \Rightarrow E(a) \in Z$ daca **$a \in \{-1, 2, 3\}$**

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