

B. ECUATII DE GRADUL I

$$1) 5(1-x) = -3(x-3) + 2x \Rightarrow 5 - 5x = -3x + 9 + 2x \Rightarrow -5x + 3x - 2x = 9 - 5 \Rightarrow -4x = 4 \cdot /(-1) \Rightarrow 4x = -4 \cdot /:(4) \Rightarrow x = -1$$

$$2) \frac{{}^3x+1}{{}^4} - \frac{{}^1x+3}{{}^{12}} = \frac{{}^25(3-x)}{{}^6} \cdot /:(12) \Rightarrow 3 \cdot (x+1) - 1 \cdot (x+3) = 10 \cdot (3-x) \Rightarrow 3x + 3 - x - 3 = 30 - 10x \Rightarrow 3x - x + 10x = 30 \Rightarrow 12x = 30 \cdot /:(12) \Rightarrow x = \frac{30}{12} \Rightarrow x = \frac{5}{2}$$

$$3) 8^{3x-2} = 16 \Rightarrow (2^3)^{3x-2} = 2^4 \Rightarrow 2^{9x-6} = 2^4 \Rightarrow 9x - 6 = 4 \Rightarrow 9x = 10 \cdot /:(9) \Rightarrow x = \frac{10}{9}$$

$$4) \overline{2X3} + \overline{13X} + \overline{X64} = 952$$

$$\overline{2X3} = 3 \cdot 1 + X \cdot 10 + 2 \cdot 100 = 3 + 10X + 200 = 10X + 203$$

$$\overline{13X} = X \cdot 1 + 3 \cdot 10 + 1 \cdot 100 = X + 30 + 100 = X + 130$$

$$\overline{X64} = 4 \cdot 1 + 6 \cdot 10 + X \cdot 100 = 4 + 60 + 100X = 100X + 64$$

$$10X + 203 + X + 130 + 100X + 64 = 952 \Rightarrow 111X + 397 = 952 \Rightarrow 111X = 555 \Rightarrow X = 5$$

$$5) 3^x + 3^{x+2} = 90 \Rightarrow 3^x + 3^x \cdot 3^2 = 90 \Rightarrow 3^x + 9 \cdot 3^x = 90 \Rightarrow 10 \cdot 3^x = 90 \Rightarrow 3^x = 9 \Rightarrow 3^x = 3^2 \Rightarrow x = 2$$

$$6) (2\sqrt{x} - 3) \cdot (4 - 3\sqrt{x}) = 22 - 6x$$

$$8\sqrt{x} - 6x - 12 + 9\sqrt{x} = 22 - 6x \Rightarrow 8\sqrt{x} + 9\sqrt{x} - 6x + 6x = 22 + 12 \Rightarrow 17\sqrt{x} = 34 \Rightarrow \sqrt{x} = 2 \Rightarrow$$

$$\Rightarrow x = 4$$

$$7) |2x-5| = 3$$

$$2x - 5 = 3$$

$$2x = 3 + 5$$

$$2x = 8 \cdot /:(2)$$

$$x = 4$$

si

$$2x - 5 = -3$$

$$2x = -3 + 5$$

$$2x = 2 \cdot /:(2)$$

$$x = 1$$

$$\Rightarrow S = \{1, 4\}$$

$$8) \sqrt{x - \sin 30^\circ} + |2x-1| + (x-0,5)^2 = 0$$

$$\sqrt{x - \sin 30^\circ} \geq 0 \text{ oricare ar fi } x \text{ numar real}$$

$$|2x - 1| \geq 0 \text{ oricare ar fi } x \text{ numar real}$$

$$(x - 0,5)^2 \geq 0 \text{ oricare ar fi } x \text{ numar real}$$

} \Rightarrow toti termenii sunt pozitivi

O suma de termeni pozitivi este zero daca si numai daca fiecare termen este zero \Rightarrow

$$x - \sin 30^\circ = 0$$

$$x = \sin 30^\circ$$

$$1$$

$$x = \frac{1}{2}$$

$$2$$

$$2x - 1 = 0$$

$$2x = 1$$

$$1$$

$$x = \frac{1}{2}$$

$$2$$

$$x - 0,5 = 0$$

$$x = 0,5$$

$$1$$

$$x = \frac{1}{2}$$

$$2$$

$$\Rightarrow S = \left\{ \frac{1}{2} \right\}$$

$$x^2 - 2\sqrt{6}$$

$$9) \frac{x^2 - 2\sqrt{6}}{10} = 0,5 \Rightarrow x^2 - 2\sqrt{6} = 5 \Rightarrow x^2 = 5 + 2\sqrt{6} \Rightarrow x_1 = +\sqrt{5 + 2\sqrt{6}} \text{ si } x_2 = -\sqrt{5 + 2\sqrt{6}}$$

$$\text{dar } \sqrt{5 + 2\sqrt{6}} = \sqrt{(\sqrt{3} + \sqrt{2})^2} = |\sqrt{3} + \sqrt{2}| = \sqrt{3} + \sqrt{2} \Rightarrow x_1 = \sqrt{3} + \sqrt{2} \text{ si } x_2 = -\sqrt{3} - \sqrt{2}$$

$$10) (x+2)^2 + (x+3) \cdot (x-3) = 2(x-1)^2 \Rightarrow x^2+4x+4 + x^2-9 = 2x^2-4x+2 \Rightarrow x^2+x^2-2x^2+4x+4x=2-4+9 \Rightarrow$$

$$\Rightarrow 8x = 7 \quad /:(8) \Rightarrow x = \frac{7}{8}$$

$$11) 0,5(x-1) - 0,75(x-2) = 1,6x \Rightarrow \frac{5^{(5)} \cdot x - 5}{2} - \frac{3^{(3)} \cdot (x-2)}{4} = \frac{16 \cdot x}{3} \quad / \cdot 12 \Rightarrow 6(x-1) - 9(x-2) = 20x \Rightarrow$$

$$0,5 = \frac{5^{(5)} \cdot 1}{10} = \frac{2}{20} \Rightarrow 6x-6-9x+18 = 20x \Rightarrow 6x-9x-20x = 6-18 \Rightarrow -23x = -12 \quad / \cdot (-1) \Rightarrow$$

$$0,75 = \frac{75^{(25)} \cdot 3}{100} = \frac{4}{100} \Rightarrow 23x = 12 \quad /:(23) \Rightarrow x = \frac{12}{23}$$

$$1,6 = \frac{16 \cdot 1}{10} = \frac{15^{(3)} \cdot 5}{90} = \frac{5}{18}$$

$$12) 2(x-3) - \sqrt{x^2-2x+1} = 3x+1$$

$$\sqrt{x^2-2x+1} = \sqrt{(x-1)^2} = |x-1| \quad ; \quad |x-1| = \begin{cases} x-1 & \text{daca } x-1 > 0 \Rightarrow x > 1 \\ 0 & \text{daca } x-1 = 0 \Rightarrow x = 1 \\ 1-x & \text{daca } x-1 < 0 \Rightarrow x < 1 \end{cases}$$

Cazul I. $|x-1| = x-1$

$$2(x-3) - (x-1) = 3x+1 \Rightarrow 2x-6-x+1 = 3x+1 \Rightarrow 2x-x-3x = 1+6-1 \Rightarrow -2x = 6 \quad / \cdot (-1) \Rightarrow 2x = -6 \Rightarrow x = -3$$

In acest caz solutia $x = -3$ nu este valabila deoarece nu este indeplinita conditia $x > 1$

Cazul II. $|x-1| = 0$

$$2(x-3) - 0 = 3x+1 \Rightarrow 2x-6 = 3x+1 \Rightarrow 2x-3x = 1+6 \Rightarrow -x = 7 \quad / \cdot (-1) \Rightarrow x = -7$$

In acest caz solutia $x = -7$ nu este valabila deoarece nu este indeplinita conditia $x = 1$

Cazul III. $|x-1| = 1-x$

$$2(x-3) - (1-x) = 3x+1 \Rightarrow 2x-6-1+x = 3x+1 \Rightarrow 2x+x-3x = 1+1+6 \Rightarrow 0 = 7 \quad (\text{F}) \Rightarrow x \in \emptyset$$

In concluzie: **ecuatia nu are solutii reale.**

Obs. Mai exista si alte metode de rezolvare, dar am utilizat aceasta metoda ca sa abordez si explicitarea modului.

$$13) \frac{2x+3}{x-1} = \frac{5}{x-1} + 8$$

ATENTIE! La acest tip de ecuatii, inainte de a incepe rezolvarea propriu-zisa, se pun conditiile de existenta a fractiilor (deoarece la numitor apare necunoscuta x)

O fractie exista (are sens) cand numitorul este diferit de zero.

Conditie: $x - 1 \neq 0 \Rightarrow x \neq 1$

$$\frac{2x+3}{x-1} = \frac{5}{x-1} + x \cdot 8 \Rightarrow 2x+3 = 5 + 8 \cdot (x-1) \Rightarrow 2x+3 = 5 + 8x-8 \Rightarrow 2x-8x = 5-8-3 \Rightarrow -6x = -6 \quad / \cdot (-1) \Rightarrow$$

$$\Rightarrow 6x = 6 \Rightarrow x = 1, \text{ dar } x \neq 1 \Rightarrow \text{ecuatia nu are solutii reale sau } x \in \emptyset$$

14) Solutia ecuatiei $|x| - 5 = -2$ in multimea $Z - N$ este

Multimea $Z-N$ reprezinta multimea numerelor **intregi negative**

$$|x| = -2 + 5 \Rightarrow |x| = 3 \Rightarrow x = 3 \text{ si } x = -3, \text{ dar } x \in Z-N \Rightarrow \text{solutia ecuatiei este } x = -3$$

15) Solutia ecuatiei $|5x - 1| = 9$ in $R - Z$ este

Multimea $R-Z$ reprezinta multimea numerelor reale care nu sunt intregi

$$|5x-1| = 9 \Rightarrow 5x-1 = 9 \Rightarrow 5x = 9+1 \Rightarrow 5x = 10 \quad /:(5) \Rightarrow x = 2$$
$$5x - 1 = -9 \Rightarrow 5x = -9+1 \Rightarrow 5x = -8 \quad /:(5) \Rightarrow x = -\frac{8}{5}$$

Deoarece $x \in R - Z \Rightarrow$ **solutia ecuatiei este $x = -\frac{8}{5}$**

16) Solutia ecuatiei $|2x + \sqrt{2}| = \sqrt{2}$ in $R - Q$ este

Multimea $R - Q$ reprezinta multimea numerelor **irrationale (radicali, numere zecimale periodice, π)**

$$|2x + \sqrt{2}| = \sqrt{2} \Rightarrow 2x + \sqrt{2} = \sqrt{2} \Rightarrow 2x = \sqrt{2} - \sqrt{2} \Rightarrow 2x = 0 \quad /:(2) \Rightarrow x = 0$$
$$\Rightarrow 2x + \sqrt{2} = -\sqrt{2} \Rightarrow 2x = -\sqrt{2} - \sqrt{2} \Rightarrow 2x = -2\sqrt{2} \quad /:(2) \Rightarrow x = -\sqrt{2}$$

Deoarece $x \in R - Q \Rightarrow$ **solutia ecuatiei este $x = -\sqrt{2}$**

17) Rezolvati in N ecuatia $(x + 2)(y - 1) = 12$

Caut perechile de numere *naturale* a caror produs sa fie $12 \Rightarrow$

$$\begin{aligned} 1 \cdot 12 &= 12 \\ 12 \cdot 1 &= 12 \\ 2 \cdot 6 &= 12 \\ 6 \cdot 2 &= 12 \\ 3 \cdot 4 &= 12 \\ 4 \cdot 3 &= 12 \end{aligned}$$

Egalez parantezele din stanga cu perechile de numere gasite \Rightarrow

$$x + 2 = 1 \Rightarrow x = 1 - 2 \Rightarrow x = -1 \quad \text{si} \quad y - 1 = 12 \Rightarrow y = 12 + 1 \Rightarrow y = 13$$

$$x + 2 = 12 \Rightarrow x = 12 - 2 \Rightarrow x = 10 \quad \text{si} \quad y - 1 = 1 \Rightarrow y = 1 + 1 \Rightarrow y = 2$$

$$x + 2 = 2 \Rightarrow x = 2 - 2 \Rightarrow x = 0 \quad \text{si} \quad y - 1 = 6 \Rightarrow y = 6 + 1 \Rightarrow y = 7$$

$$x + 2 = 6 \Rightarrow x = 6 - 2 \Rightarrow x = 4 \quad \text{si} \quad y - 1 = 2 \Rightarrow y = 2 + 1 \Rightarrow y = 3$$

$$x + 2 = 3 \Rightarrow x = 3 - 2 \Rightarrow x = 1 \quad \text{si} \quad y - 1 = 4 \Rightarrow y = 4 + 1 \Rightarrow y = 5$$

$$x + 2 = 4 \Rightarrow x = 4 - 2 \Rightarrow x = 2 \quad \text{si} \quad y - 1 = 3 \Rightarrow y = 3 + 1 \Rightarrow y = 4$$

Solutiile ecuatiei se exprima astfel:

$(x,y) \in \{(10;2), (0;7), (4;3), (1;5), (2;4)\}$ * Perechea $(-1;13)$ nu este valabila deoarece $-1 \notin N$

18) Rezolvati in Z ecuatia $(x+3)(y-2) = 15$

Caut perechile de numere *intregi* a caror produs sa fie $15 \Rightarrow$ perechile $(1;15), (-1;-15), (15;1), (-15,-1), (3;5), (-3;-5), (5;3), (-5;-3)$. Se continua rezolvarea ca la exercitiul 17.