M. CALCULE ALGEBRICE - PROBLEME REZOLVATE

a) Efectuați calculele:

1) $(5x-3)^2 - 6(2x+3)(2x-3) - x(x+1)$

REZOLVARE

$(5x)^2 - 2\cdot 5x\cdot 3 + 3^2 - 6(4x^2 - 9) - x^2 - x = \underline{25x^2} \underline{- 30x} + 9 \underline{- 24x^2} + 54 \underline{- x^2} \underline{- x} =$ **$-31x + 63$**

2) $3(x-\sqrt{5})^2 - (\sqrt{3}x+1)(\sqrt{3}x-1) + 6\sqrt{5}x$

REZOLVARE

$3(x^2 - 2\cdot x\cdot\sqrt{5} + \sqrt{5^2}) - [(\sqrt{3}x)^2 - 1^2] + 6\sqrt{5}x = \underline{3x^2} \underline{- 6\sqrt{5}x} + 15 \underline{- 3x^2} + 1 \underline{+ 6\sqrt{5}x} =$ **16**

b) Descompuneti in factori:

1) $4x^2(2x-1) + 16(1-2x)$

REZOLVARE

$4x^2(2x-1) - 16(2x-1) = 4(2x-1)(x^2-4) =$ **$4(2x-1)(x-2)(x+2)$**

2) $x^3 - 5x^2 - 2x + 10$

REZOLVARE

$x^2(x - 5) - 2(x - 5) = (x - 5)(x^2 - 2) =$ **$(x - 5)(x - \sqrt{2})(x + \sqrt{2})$**

3) $x^3 - 8x^2 + 19x - 12$

REZOLVARE

Verific care din divizorii termenului liber (12) este solutie a polinomului.

Pentru $x=1 \Rightarrow 1^3 - 8\cdot 1^2 + 19\cdot 1 - 12 = 1 - 8 + 19 - 12 = 0 \Rightarrow$ **1** este solutie a polinomului $\Rightarrow$

$\Rightarrow (x - 1)$ trebuie sa fie factor comun. Scot "fortat" pe $(x - 1)$ factor comun astfel:

$x^3 - \underbrace{x^2 - 7x^2}_{-8x^2} + \underbrace{7x + 12x}_{+19x} - 12 = x^2(x - 1) - 7x(x - 1) + 12(x - 1) = (x - 1)(x^2 - 7x + 12)$

$x^2 - 7x + 12 = x^2 - 3x - 4x + 12 = x(x - 3) - 4(x - 3) = (x - 3)(x - 4)$

(metoda de descompunere de mai sus este explicata in **Algebra teorie - Calcule algebrice - 6.1**)

$x^3 - 8x^2 + 19x - 12 =$ $(x - 1)(x - 3)(x - 4)$

4) $16x^4 - 81y^4$

REZOLVARE

$(4x^2)^2 - (9y^2)^2 = (4x^2 - 9y^2)(4x^2 + 9y^2) = [(2x)^2 - (3y)^2](4x^2 + 9y^2) =$ **$(2x - 3y)(2x + 3y)(4x^2 + 9y^2)$**

5) $(x^2 - 4)^2 - (x + 2)^2$

REZOLVARE

$[(x^2 - 4) - (x + 2)][(x^2 - 4) + (x + 2)] = (x^2 - 4 - x - 2)(x^2 - 4 + x + 2) = (x^2 - x - 6)(x^2 + x - 2)$

$x^2 - x - 6 = x^2 - 3x + 2x - 6 = x(x - 3) + 2(x - 3) = (x - 3)(x + 2)$

$x^2 + x - 2 = x^2 + 2x - x - 2 = x(x + 2) - (x + 2) = (x + 2)(x - 1)$

$(x^2 - 4)^2 - (x + 2)^2 =$ $(x - 3)(x + 2)\ (x + 2)(x - 1) =$ **$(x - 1)(x - 3)(x + 2)^2$**

Metoda2: $(x-2)^2(x+2)^2 - (x+2)^2 = (x+2)^2[(x-2)^2 - 1] = (x+2)^2(x-2 - 1)(x-2 + 1) = (x+2)^2(x-3)(x-1)$

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6) $36x^2 + 12x + 1$

REZOLVARE

Verific daca expresia se poate restrange ca un patrat perfect

$36x^2 + 12x + 1$

$\downarrow \qquad \downarrow$

$(6x)^2 \qquad (1)^2$

$2 \cdot 6x \cdot 1 = 12x \Rightarrow$ **$36x^2 + 12x + 1 = (6x + 1)^2$**

7) $x^2 - 2\sqrt{3}x + 3$

REZOLVARE

$x^2 - 2\sqrt{3}x + 3$

$\downarrow \qquad \downarrow$

$(x)^2 \qquad (\sqrt{3})^2$

$2 \cdot x \cdot \sqrt{3} = 2\sqrt{3}x \Rightarrow$ **$x^2 - 2\sqrt{3}x + 3 = (x + \sqrt{3})^2$**

8) $4x^2 + 4x - 3$

REZOLVARE

Utilizez metoda prezentata in **Algebra teorie - Calcule algebrice - 6.1**

$4 \cdot (-3) = -12 \Rightarrow a \cdot b = -12$ si $a+b = +4 \Rightarrow a = 6$ si $b = -2 \Rightarrow$ **$4x = 6x - 2x \Rightarrow$**

$4x^2 + 6x - 2x - 3 = 2x(2x + 3) - (2x + 3) =$ **$(2x + 3)(2x - 1)$**

9) $6x^2 - 11x + 3$

REZOLVARE

$6 \cdot (+3) = +18 \Rightarrow a \cdot b = +18$ si $a+b = -11 \Rightarrow a = -9$ si $b = -2 \Rightarrow$ **$-11x = -9x - 2x \Rightarrow$**

$6x^2 - 9x - 2x + 3 = 3x(2x - 3) - (2x - 3) =$ **$(2x - 3)(3x - 1)$**

10) $(x^2-1)^2 - 5(x^2-1) + 6$

REZOLVARE

***Notez* $x^2 - 1 = a \Rightarrow (x^2-1)^2 - 5(x^2-1) + 6 = a^2 - 5a + 6 \Rightarrow$**

$a^2 - 5a + 6 = a^2 - 2a - 3a + 6 = a(a - 2) - 3(a - 2) = (a - 2)(a - 3)$

Inlocuiesc pe **a** cu **$x^2 - 1$** $\Rightarrow (x^2 - 1 - 2)(x^2 - 1 - 3) = (x^2 - 3)(x^2 - 4) =$ **$(x - \sqrt{3})(x + \sqrt{3})(x - 2)(x + 2)$**

11) $x^4 - 6x^2 - 16$

REZOLVARE

***Notez* $x^2 = a \Rightarrow x^4 = a^2 \Rightarrow x^4 - 6x^2 - 16 = a^2 - 6a - 16 \Rightarrow$**

$a^2 - 6a - 16 = a^2 - 8a + 2a - 16 = a(a - 8) + 2(a - 8) = (a - 8)(a + 2)$

Inlocuiesc pe **a** cu **x^2** $\Rightarrow (x^2 - 8)(x^2 + 2) =$ **$(x - 2\sqrt{2})(x + 2\sqrt{2})(x^2 + 2)$**

12) $x(x+1)(x^2+x+2) + 1$

REZOLVARE

In primul rand efectuez **x(x+1)** $\Rightarrow x(x+1)(x^2+x+2) + 1 = (x^2 + x)(x^2 + x + 2) + 1$

***Notez* $x^2 + x = a \Rightarrow (x^2 + x)(x^2 + x + 2) + 1 = a(a + 2) + 1 \Rightarrow$**

$a(a + 2) + 1 = a^2 + 2a + 1 = (a + 1)^2$

Inlocuiesc pe **a** cu **$x^2 + x$** $\Rightarrow (x^2 + x + 1)^2 \Rightarrow x(x+1)(x^2+x+2) + 1 =$ **$(x^2 + x + 1)^2$** .

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