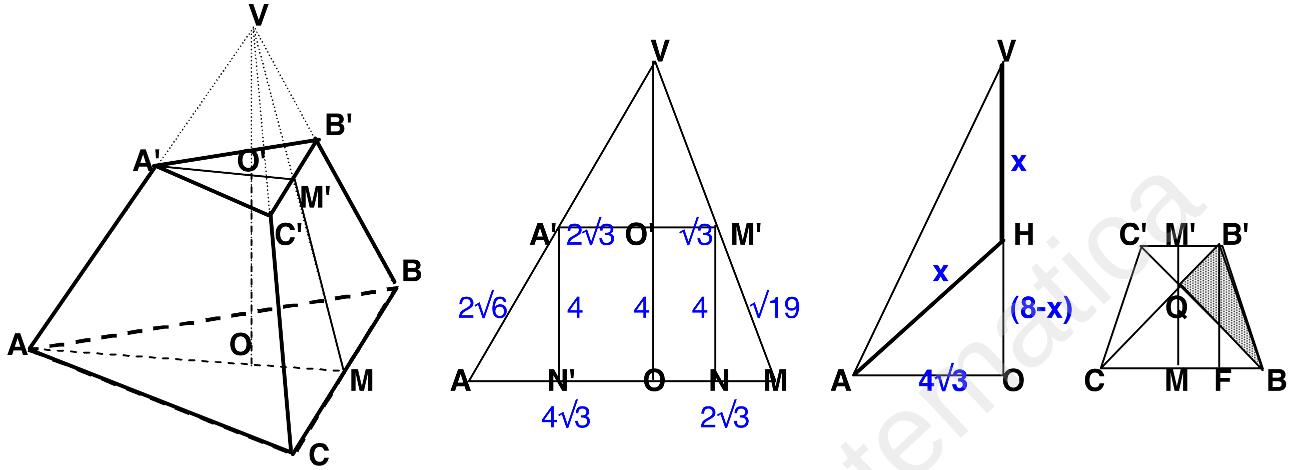
J2. TRUNCHIUL DE PIRAMIDĂ TRIUNGHIULARA REGULATA - PROBLEME REZOLVATE

1. Un trunchi de piramidă triunghiulară regulată ABCA'B'C' are perimetrele bazelor de 18 cm respectiv 36 cm și inălțimea OO' este doua treimi din latura bazei mici.Se cere:
a) muchia laterala b)Al, At , V trunch. c) Al și V piramidei din care provine trunchiul
d) Aria ΔBQB' unde Q este punctul de intersecție a segmentelor BC' și B'C
e) Daca punctul H este situat la distanta egala fata de punctele A,B,C,V(virful piramidei din care provine trunchiul), calculati lungimea segmentului VH.

REZOLVARE



a) $P_b=18 \Rightarrow 3\cdot A'B'=18 \Rightarrow$ **$A'B'=6cm$**; $P_B=36 \Rightarrow 3\cdot AB=36 \Rightarrow$ **$AB=12cm$**; $OO'=2/3\cdot A'B' \Rightarrow$ **$OO'=4cm$**

In ΔABC echilateral, $AM=\dfrac{AB\sqrt{3}}{2} \Rightarrow$ **$AM=6\sqrt{3}cm$** ; $OM=\dfrac{1}{3}\cdot AM \Rightarrow$ **$OM=2\sqrt{3}cm$** ; **AO**$=2\cdot OM=$**$4\sqrt{3}cm$**

In ΔA'B'C' $A'M'=\dfrac{A'B'\sqrt{3}}{2} \Rightarrow$ **$A'M'=3\sqrt{3}cm$**; $O'M'=\dfrac{1}{3}\cdot A'M' \Rightarrow$ **$O'M'=\sqrt{3}cm$** ; **$A'O'$**$=2\cdot O'M'=$**$2\sqrt{3}cm$**

In ΔM'NM, $\angle N=90° \Rightarrow M'M^2=M'N^2+NM^2=16+3=19 \Rightarrow$ **$M'M=\sqrt{19}$ cm**

In ΔA'N'A, $\angle N'=90° \Rightarrow A'A^2=A'N'^2+N'A^2=16+8=24 \Rightarrow$ **$A'A=2\sqrt{6}$ cm** .

b) **$Al=\dfrac{(P_B+Pb)\cdot a_{tr}}{2}=\dfrac{(36+18)\cdot\sqrt{19}}{2}=$** **$27\sqrt{19}cm^2$** ; **$A_B$**$=\dfrac{AB^2\sqrt{3}}{4}=\dfrac{144\sqrt{3}}{4}=$**$36\sqrt{3}cm^2$**; **$Ab$**$=\dfrac{36\sqrt{3}}{4}=$**$9\sqrt{3}cm^2$**

At$=Al+A_B+Ab=$**$(27\sqrt{19}+45\sqrt{3})cm^2$** ; **$V=$**$\dfrac{OO'}{3}(A_B+Ab+\sqrt{A_B\cdot Ab})=\dfrac{4}{3}(36\sqrt{3}+9\sqrt{3}+18\sqrt{3})=$ **$84\sqrt{3}$ cm^3**.

c) Deoarece $A'M' = AM / 2 \Rightarrow A'M'$= linie mijlocie in ΔVAM $\Rightarrow$ **$VO=$**$2\cdot O'O=$**$8cm$** ; **VM**$=2\cdot M'M=$**$2\sqrt{19}cm$**

$Al_{PIRAMIDA}=$ $\dfrac{P_B\cdot VM}{2}=\dfrac{36\cdot 2\sqrt{19}}{2}=$**$36\sqrt{19}cm^2$** ; **$V_{PIRAMIDA}=$**$\dfrac{A_B\cdot VO}{3}=\dfrac{36\sqrt{3}\cdot 8}{3}=$ **$96\sqrt{3}$ cm^3**.

d) Aria $\Delta BQB'$ = Aria $\Delta BCB'$ - Aria ΔBCQ ; $\Delta CQM \sim \Delta CB'F \Rightarrow \dfrac{QM}{B'F} = \dfrac{CM}{CF} \Rightarrow \dfrac{QM}{\sqrt{19}} = \dfrac{6}{9} \Rightarrow$ $\mathbf{QM = \dfrac{2\sqrt{19}}{3}}$ **cm**

Aria $\Delta BCB' = \dfrac{B'F \cdot BC}{2} = \dfrac{\sqrt{19} \cdot 12}{2} = 6\sqrt{19}\text{cm}^2$, Aria $\Delta BCQ = \dfrac{QM \cdot BC}{2} = \dfrac{2\sqrt{19}}{3} \cdot \dfrac{12}{2} = 4\sqrt{19}\text{cm}^2 \Rightarrow$

Aria $\Delta BQB'$ = $6\sqrt{19} - 4\sqrt{19}$ **= $2\sqrt{19}$ cm^2**

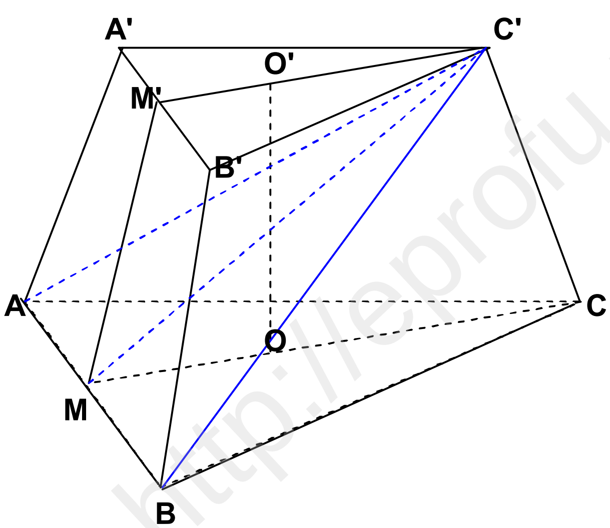
e) Daca $HA=HB=HC \Rightarrow$ H este pe directia centrului cercului circumscris ΔABC deci pe inaltimea VO

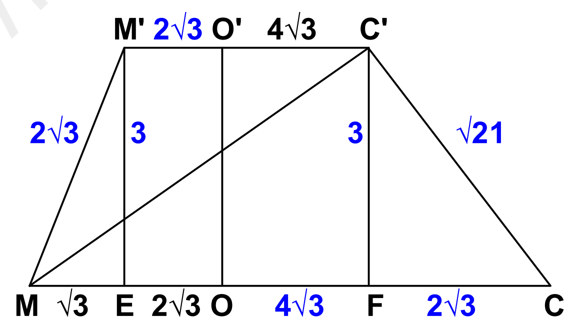
Notam $VH=HA=x$; In ΔHOA , $AO=4\sqrt{3}$, $HO=VO-x=8-x \Rightarrow HA^2=AO^2+HO^2 \Rightarrow x^2=48+(8-x)^2$

$\Rightarrow x^2 = 48+64-16x+x^2 \Rightarrow x^2+16x-x^2=48+64 \Rightarrow 16x=112 \Rightarrow x=7 \Rightarrow$ **VH=7 cm**

2. Un trunchi de piramida triunghiulara regulata ABCA'B'C' are raza cercului circumscris bazei mici de $4\sqrt{3}$ cm, raza cercului inscris bazei mari de $3\sqrt{3}$ cm si unghiul dintre inaltimea trunchiului si o fața laterala a acestuia de 30°. Punctul M este mijlocul muchiei AB. Se cere:
a) Aria laterala, aria totala si volumul trunchiului.
b) Distanta de la punctul M la planul (BB'C').
c) Tangenta unghiului dintre planele (AC'B) si (ABC).
d) Distanta de la mijlocul inaltimii trunchiului la o fata laterala a acestuia.

REZOLVARE





a) Raza cercului circumscris bazei mici este segmentul $O'C' \Rightarrow$ **$O'C' = 4\sqrt{3}$ cm**

Raza cercului inscris bazei mici este segmentul $O'M' = \dfrac{O'C'}{2} = \dfrac{4\sqrt{3}}{2} \Rightarrow$ **$O'M' = 2\sqrt{3}$ cm**

Raza cercului inscris bazei mari este segmentul $OM \Rightarrow$ **$OM = 3\sqrt{3}$ cm**

Raza cercului circumscris bazei mari este segmentul $OC = 2 \cdot OM = 2 \cdot 3\sqrt{3} \Rightarrow$ **$OC = 6\sqrt{3}$ cm**

(Relatiile dintre raza cercului inscris si circumscris unui triunghi echilateral sunt prezentate in detaliu la capitolul triunghiuri)

Inaltimea bazei mici $M'C' = O'C' + O'M' = 4\sqrt{3} + 2\sqrt{3} = 6\sqrt{3}$ cm $\Rightarrow \dfrac{l\sqrt{3}}{2} = 6\sqrt{3} \Rightarrow l\sqrt{3} = 12\sqrt{3} \Rightarrow$ **$l = 12$ cm**

Inaltimea bazei mari $MC = OC + OM = 6\sqrt{3} + 3\sqrt{3} = 9\sqrt{3}$ cm $\Rightarrow \dfrac{L\sqrt{3}}{2} = 9\sqrt{3} \Rightarrow L\sqrt{3} = 18\sqrt{3} \Rightarrow$ **$L = 18$ cm**

Perimetrul bazei mici $\mathbf{P_b} = 3\cdot l = 3\cdot 12 =$ **36 cm** ; Aria bazei mici $\mathbf{A_b} = \dfrac{l^2\sqrt{3}}{4} = \dfrac{144\sqrt{3}}{4} = \mathbf{36\sqrt{3}\ cm^2}$

Perimetrul bazei mari $\mathbf{P_B} = 3\cdot L = 3\cdot 18 =$ **54 cm** ; Aria bazei mari $\mathbf{A_B} = \dfrac{L^2\sqrt{3}}{4} = \dfrac{324\sqrt{3}}{4} = \mathbf{81\sqrt{3}\ cm^2}$

$\angle(O'O; (ABB'A')) = \angle(M'E; M'M) = \angle(MM'E) = 30°$

In $\Delta M'EM$ dr. in E, daca $m\angle(MM'E) = 30° \Rightarrow ME = \dfrac{M'M}{2} \Rightarrow \mathbf{M'M} = 2\cdot ME = \mathbf{2\sqrt{3}\ cm}$

In $\Delta M'EM$ dr. in E $\Rightarrow M'E^2 = M'M^2 - ME^2 = (2\sqrt{3})^2 - (\sqrt{3})^2 = 12 - 3 = 9 \Rightarrow M'E = \sqrt{9} \Rightarrow$ **$M'E = 3$ cm.**

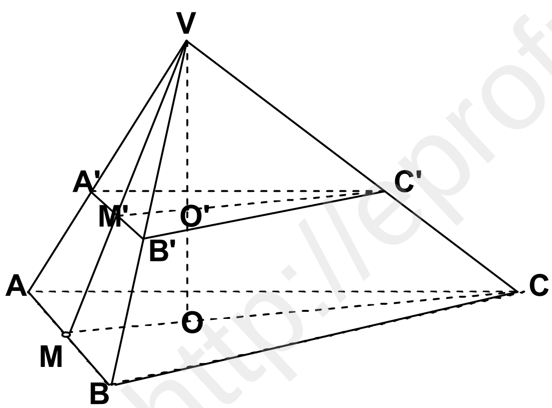
Aria laterala a trunchiului $= \dfrac{(\mathbf{P_B} + \mathbf{P_b})\cdot \mathbf{a_{tr}}}{\mathbf{2}} = \dfrac{(54 + 36)\cdot 2\sqrt{3}}{2} \Rightarrow$ **Aria laterala $= 90\sqrt{3}$ cm^2**

Aria totala = Al + $\mathbf{A_B}$ + $\mathbf{A_b}$ $= 90\sqrt{3} + 81\sqrt{3} + 36\sqrt{3} \Rightarrow$ **Aria totala $= 207\sqrt{3}$ cm^2**

Volumul trunchiului $= \dfrac{\mathbf{h}}{\mathbf{3}}(\mathbf{A_B} + \mathbf{A_b} + \sqrt{\mathbf{A_B}\cdot\mathbf{A_b}}) = \dfrac{3}{3}(81\sqrt{3} + 36\sqrt{3} + \sqrt{81\sqrt{3}\cdot 36\sqrt{3}}) = 117\sqrt{3} + 54\sqrt{3}$

Volumul $=171\sqrt{3}$ cm^3 .

b)



$d(M; (BB'C')) = d(M; (BB'C'C)) = d(M; (VBC))$

Formez piramida MVBC si scriu volumul in 2 moduri

$$V_{MVBC} = \frac{\text{Aria } \Delta VBC \cdot d(M; (VBC))}{3} = \frac{\text{Aria } \Delta MBC\cdot VO}{3}$$

$$\mathbf{d(M; (VBC)) = \frac{Aria\ \Delta MBC \cdot VO}{Aria\ \Delta VBC}}$$

Calculez VM si VO din asemanarea triunghiurilor VO'M' si VOM

$M'O' \parallel MO \Rightarrow \Delta VO'M' \sim \Delta VOM \Rightarrow \dfrac{VO'}{VO} = \dfrac{O'M'}{OM} = \dfrac{VM'}{VM} \Rightarrow \dfrac{VO - 3}{VO} = \dfrac{2\sqrt{3}}{3\sqrt{3}} = \dfrac{VM - 2\sqrt{3}}{VM} \Rightarrow$

$3\cdot(VO - 3) = 2VO \Rightarrow 3VO - 9 = 2VO \Rightarrow 3VO - 2VO = 9 \Rightarrow$ **$VO = 9$ cm**

$3(VM - 2\sqrt{3}) = 2VM \Rightarrow 3VM - 6\sqrt{3} = 2VM \Rightarrow 3VM - 2VM = 6\sqrt{3} \Rightarrow$ **$VM = 6\sqrt{3}$ cm**

$$\textbf{Aria } \Delta MBC = \frac{\textbf{Aria } \Delta ABC}{2} = \frac{81\sqrt{3}}{2} \textbf{ cm}^2 \text{ ; } \textbf{Aria } \Delta VBC = \text{Aria } \Delta VAB = \frac{AB \cdot VM}{2} = \frac{18 \cdot 6\sqrt{3}}{2} = \mathbf{54\sqrt{3}\ cm^2}$$

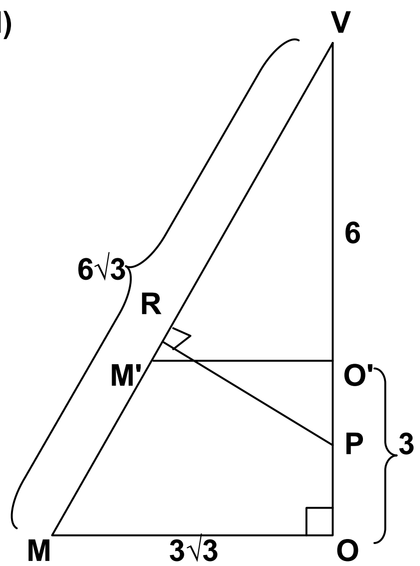
$$d(M; (VBC)) = \frac{81\sqrt{3}}{2} \cdot \frac{9}{1} \cdot \frac{1}{54\sqrt{3}} = \frac{9}{2} \cdot \frac{9}{1} \cdot \frac{1}{6} = \frac{27}{4} \Rightarrow \mathbf{d(M; (VBC)) = \frac{27}{4}\ cm}\ .$$

c) tg ∠((AC'B);(ABC)).

$$\left.\begin{array}{l} (AC'B) \cap (ABC) = AB \\ C'M \perp AB;\ C'M \subset (AC'B) \\ CM \perp AB;\ CM \subset (ABC) \end{array}\right\} \Rightarrow \angle((AC'B); (ABC)) = \angle(C'M; CM) = \angle(C'MC)$$

In Δ C'FM dr. in F $\Rightarrow$ $\text{tg}\ \angle(C'MF) = \frac{C'F}{MF} = \frac{3}{7\sqrt{3}} = \frac{3\sqrt{3}}{7 \cdot 3} = \frac{\sqrt{3}}{7} \Rightarrow \mathbf{tg\ \angle(C'MC) = \frac{\sqrt{3}}{7}}$

d)



Punctul P este mijlocul inaltimii OO'

d(P; (ABB'A')) = d(P; (VAB)) = d(P; VM)

Construiesc PR ⊥ VM ⇒ **d(P; VM) = PR**

Deoarece **m∠(PVR) = m∠(OVM)** si **m∠(VRP) = m∠(VOM)** ⇒

$$\Delta VRP \sim \Delta VOM \Rightarrow \frac{PR}{OM} = \frac{VP}{VM} \Rightarrow \mathbf{PR = \frac{OM \cdot VP}{VM}}$$

$$PR = \frac{3\sqrt{3}}{1} \cdot \frac{15}{2} \cdot \frac{1}{6\sqrt{3}} = \frac{15}{4} \Rightarrow \mathbf{PR = \frac{15}{4}\ cm}$$