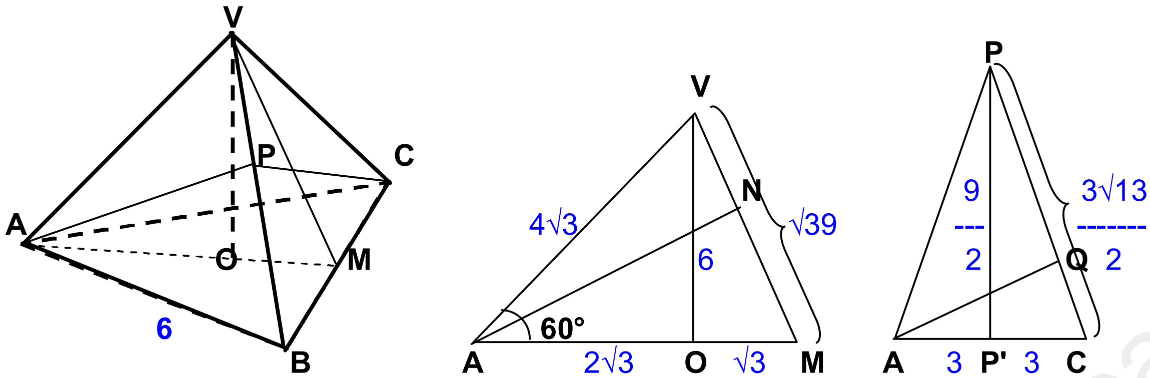
I2. PIRAMIDA TRIUNGHIULARA REGULATA - PROBLEME REZOLVATE

1) Piramida triunghiulara VABC are aria bazei $9\sqrt{3}cm^2$ si unghiul dintre o muchie laterala si planul bazei de 60°. Se cere: a) Al, At, V ; b) Distanta de la un virf al bazei la fata laterala opusa virfului : c)sinusul unghiului dintre doua fete laterale



a) $A_l=\dfrac{P_b\cdot a_p}{2}$; $A_t=A_l+A_b$; $A_b=\dfrac{l^2\sqrt{3}}{4}$; $V=\dfrac{A_b\cdot h}{3}$; $AM=\dfrac{l\sqrt{3}}{2}$; $OM=\dfrac{1}{3}\cdot AM=\dfrac{l\sqrt{3}}{6}$; $AO=2\cdot OM$

$\dfrac{l^2\sqrt{3}}{4}=9\sqrt{3} \Rightarrow l^2\sqrt{3}=36\sqrt{3} \Rightarrow l^2=36 \Rightarrow l=6 \Rightarrow$ **$AB=6cm$** ; **$OM=\dfrac{6\sqrt{3}}{6}=\sqrt{3}cm$** ; **$AO=2\sqrt{3}cm$**

Proiectia lui AV pe (ABC) este AO $\Rightarrow \angle(AV;(ABC))=\angle(AV;AO)=\angle(VAO)=60°$

In ΔVOA, $\angle O=90°$, $\angle A=60° \Rightarrow \angle V=30° \Rightarrow AO=\dfrac{VA}{2} \Rightarrow VA=2\cdot AO=2\cdot 2\sqrt{3} \Rightarrow$ **$VA=4\sqrt{3}cm$**

In ΔVOA, $\angle O=90° \Rightarrow VO^2=VA^2-AO^2=(4\sqrt{3})^2-(2\sqrt{3})^2=48-12=36 \Rightarrow$ **$VO=6cm$**

In ΔVOM, $\angle O=90° \Rightarrow VM^2=VO^2+OM^2=6^2+(\sqrt{3})^2=36+3=39 \Rightarrow$ **$VM=\sqrt{39}cm$**

$A_l=\dfrac{3\cdot 6\cdot\sqrt{39}}{2}=9\sqrt{39}cm^2$; **$A_t=9\sqrt{39}+9\sqrt{3}=9\sqrt{3}(\sqrt{13}+1)cm^2$** ; **$V=\dfrac{9\sqrt{3}\cdot 6}{3}=18\sqrt{3}cm^3$** .

b) $(VAM)\perp(VBC)$ - VM = latura comuna $\Rightarrow d(A;(VBC))=$ **$d(A;VM)=AN$**, unde $(AN\perp VM)$

In $\Delta VAM \Rightarrow AN\cdot VM=VO\cdot AM \Rightarrow AN=\dfrac{VO\cdot AM}{VM}=\dfrac{6\cdot 3\sqrt{3}}{\sqrt{39}}=\dfrac{18}{\sqrt{13}} \Rightarrow$ **$AN=\dfrac{18\sqrt{13}}{13}$ cm**

c) $\left.\begin{matrix}(VAB)\cap(VBC)=VB\\ AP\perp VB;\ AP\subset(VAB)\\ PC\perp VB;\ PC\subset(VBC)\end{matrix}\right\} \Rightarrow$ **$\angle((VAB);(VBC))$** $=\angle(AP;PC)=$ **$\angle(APC)$**

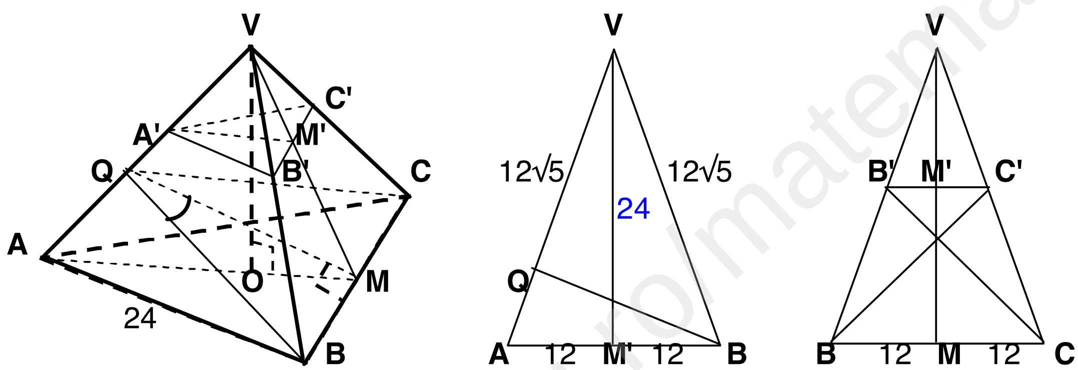
In $\Delta VBC \Rightarrow PC\cdot VB=VM\cdot BC \Rightarrow PC=\dfrac{VM\cdot BC}{VB}=\dfrac{\sqrt{39}\cdot 6}{4\sqrt{3}}=\dfrac{3\sqrt{13}}{2} \Rightarrow$ **$PC=\dfrac{3\sqrt{13}}{2}$ cm**

In $\Delta PP'C$, $\angle P'=90° \Rightarrow P'P^2 = PC^2 - P'C^2 = \dfrac{9\cdot13}{4} - 9 = \dfrac{9\cdot13 - 9\cdot4}{4} = \dfrac{9\cdot9}{4} \Rightarrow$ **$P'P = \dfrac{9}{2}$ cm**

In $\Delta PAC \Rightarrow AQ\cdot PC = PP'\cdot AC \Rightarrow AQ = \dfrac{PP'\cdot AC}{PC} = \dfrac{9}{2}\cdot\dfrac{6}{1}\cdot\dfrac{2}{3\sqrt{13}} = \dfrac{18\sqrt{13}}{13} \Rightarrow$ **$AQ = \dfrac{18\sqrt{13}}{13}$ cm**

In ΔPQA, $\angle Q=90° \Rightarrow \sin(\angle APQ) = \dfrac{AQ}{AP} = \dfrac{18\sqrt{13}}{13}\cdot\dfrac{2}{3\sqrt{13}} \Rightarrow$ **$\sin(\angle APQ) = \dfrac{12}{13}$**

2. Fie o piramida triunghiulara regulata de virf V si baza ABC. Un plan paralel cu planul (ABC) intersecteaza muchiile VA, VB, VC in punctele A', B', C' astfel incit dreptele BC' si CB' sa fie perpendiculare. Se stie ca AB=24cm si VA=12√5 cm. a)Calculati volumul piramidei VABC; b) Fie M mijlocul laturii BC. Calculati valoarea sinusului unghiului diedru dintre planele (AVM) si (AVB);c)Calculati aria laterala a trunchiului de piramida ABCA'B'C



a) **Volumul = $\dfrac{A_b\cdot h}{3}$** ; **$A_b = \dfrac{l^2\sqrt{3}}{4} \Rightarrow A_b = \dfrac{24\cdot24\cdot\sqrt{3}}{4} = 144\sqrt{3}\text{cm}^2$** **AM** $= \dfrac{l\sqrt{3}}{2} = \dfrac{24\sqrt{3}}{2} =$ **$12\sqrt{3}$cm**

$AO = \dfrac{2}{3}\cdot AM = \dfrac{2}{3}\cdot 12\sqrt{3} = 8\sqrt{3}$ cm. In ΔVOA, $\angle O=90° \Rightarrow VO^2 = VA^2 - AO^2 = 720 - 192 = 528 \Rightarrow$ **$VO=4\sqrt{33}$cm**

In ΔVMB, $\angle M=90° \Rightarrow VM^2 = VB^2 - MB^2 = 720 - 144 = 576 \Rightarrow$ **VM=24 cm** $\Rightarrow$ **VM' = 24 cm**

Volumul $= \dfrac{144\sqrt{3}\cdot4\sqrt{33}}{3}$ **$= 576\sqrt{11}\text{cm}^3$**

b) $\left.\begin{array}{l} BQ \perp VA \\ CQ \perp VA \\ BQ, CQ \subset (BQC) \end{array}\right\} \Rightarrow VA \perp (BQC)$ dar $MQ \subset (BQC) \Rightarrow VA \perp MQ \Rightarrow$ **$MQ \perp VA$**

$\left.\begin{array}{l} VM\perp BC \\ AM\perp BC \\ VM, AM \subset (AVM) \end{array}\right\} \Rightarrow BC\perp(AVM)$ dar $MQ \subset (AVM) \Rightarrow BC \perp MQ \Rightarrow$ **$MQ \perp BC \Rightarrow \Delta QMB$ dr. in M**

$$\left.\begin{array}{l}(AVM)\cap(AVB) = VA\\ MQ\perp VA;\ MQ\subset(AVM)\\ BQ\perp VA;\ BQ\subset(AVB)\end{array}\right\}\Rightarrow\angle((AVM);(AVB))=\angle(MQ;BQ)=\angle(MQB)$$

$$\text{In}\Delta VAB\Rightarrow QB\cdot VA=VM'\cdot AB \Rightarrow QB=\frac{VM'\cdot AB}{VA}=\frac{24\cdot 24}{12\sqrt{5}}\Rightarrow \mathbf{QB=\frac{48\sqrt{5}}{5}\ cm}$$

$$\text{In } \Delta QMB,\ \angle M=90^\circ \Rightarrow \sin(\angle MQB)=\frac{BM}{QB}=\frac{12}{1}\cdot\frac{5}{48\sqrt{5}}\Rightarrow \sin(\angle MQB)=\frac{\sqrt{5}}{4}$$

c) Aria laterala a trunchiului= $\dfrac{\text{(perimetrul bazei mari+perimetrul bazei mici)}\cdot\text{apotema trunch.}}{2}$

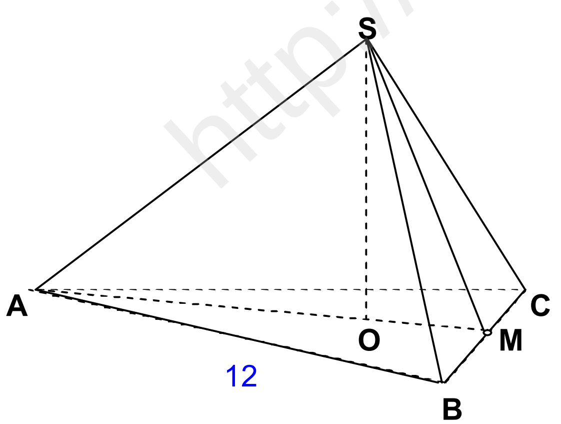
$$\text{In trapezul } BB'C'C,\ BC'\perp CB'\Rightarrow MM'=\frac{BC+B'C'}{2}\Rightarrow BC+B'C' = 2\cdot MM'$$

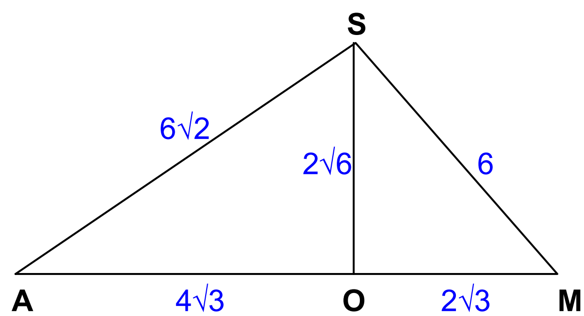
$$B'C'\parallel BC\Rightarrow\Delta VB'C'\sim\Delta VBC\Rightarrow \frac{B'C'}{BC}=\frac{VM'}{VM}\Rightarrow\frac{B'C'+BC}{BC}=\frac{VM'+VM}{VM}\Rightarrow\frac{2\cdot MM'}{24}=\frac{VM-MM'+VM}{24}$$

$2\cdot MM' = 48 - MM' \Rightarrow 3\cdot MM'=48 \Rightarrow$ **MM'=16cm ,** dar $B'C' =2\cdot MM' - BC\Rightarrow$ **B'C' = 2·16 - 24 = 8cm**

$$\textbf{Aria laterala a trunchiului} = \frac{(3\cdot 24 + 3\cdot 8)\cdot 16}{2} = \mathbf{768cm^2}$$

3. O piramida triunghiulara regulata SABC are suma muchiilor laterale 18√2 cm iar unghiul dintre muchia SA si fața (SBC) este 90°. Punctul M este mijlocul muchiei BC. Se cere:
a) Aria laterala, aria totala si volumul piramidei ; b) Masura unghiului dintre planele (SAM) si (SAB) ; c) Distanta de la punctul M la planul (SAB).





$$\textbf{a)}\ \text{Muchia laterala} = \frac{\text{suma muchiilor laterale}}{3}\Rightarrow SA = \frac{18\sqrt{2}}{3}\Rightarrow \mathbf{SA = 6\sqrt{2}\ cm}$$

$\left.\begin{array}{l} AM \perp BC \\ S, M \in (SBC) \end{array}\right\} \Rightarrow$ SM este proiectia lui SA pe (SBC) $\Rightarrow \angle(SA ; (SBC)) = \angle(SA; SM) = \angle(ASM)$

Deoarece m $\angle(ASM) = 90^\circ \Rightarrow \Delta SMA$ este dreptunghic

Deoarece SO este inaltime in piramida $\Rightarrow OM = \frac{1}{3} \cdot AM$ si $OA = \frac{2}{3} \cdot AM \Rightarrow OA = 2 \cdot OM$

In Δ SMA dr. notez OM = x $\Rightarrow$ OA = 2x si AM = 3x. Aplic ***teorema catetei*** $\Rightarrow$ $\mathbf{SA^2 = AO \cdot AM} \Rightarrow$
$\Rightarrow (6\sqrt{2})^2 = 2x \cdot 3x \Rightarrow 6x^2 = 72 \Rightarrow x^2 = 12 \Rightarrow x = \sqrt{12} \Rightarrow x = 2\sqrt{3} \Rightarrow$ **OM = 2√3 cm**, **OA = 4√3cm**

In ΔSMA dr. aplic ***teorema inaltimii*** $\Rightarrow$ $\mathbf{SO^2 = OA \cdot OM} \Rightarrow SO^2 = 4\sqrt{3} \cdot 2\sqrt{3} \Rightarrow SO^2 = 24 \Rightarrow$ **SO =2√6 cm**

In ΔSOM dr. $\Rightarrow SM^2 = SO^2 + OM^2 \Rightarrow SM^2 = (2\sqrt{6})^2 + (2\sqrt{3})^2 = 24+12=36 \Rightarrow SM=\sqrt{36} \Rightarrow$ **SM = 6 cm**

In ΔABC echilateral cu AM inaltime $\Rightarrow AM = \frac{l\sqrt{3}}{2} \Rightarrow 6\sqrt{3} = \frac{l\sqrt{3}}{2} \Rightarrow l\sqrt{3} = 12\sqrt{3} \Rightarrow l = 12 \Rightarrow$ **AB = 12 cm**

Aria bazei $= \frac{\mathbf{l^2\sqrt{3}}}{\mathbf{4}} = \frac{12^2 \cdot \sqrt{3}}{4} = \frac{144 \cdot \sqrt{3}}{4} = 36\sqrt{3}$ cm^2 $\Rightarrow$ **Aria bazei = 36√3 cm²**

Aria laterala $= \frac{\mathbf{Pb \cdot ap}}{\mathbf{2}} \Rightarrow$ Aria laterala $= \frac{3 \cdot 12 \cdot 6}{2} = 108$cm^2 $\Rightarrow$ **Aria laterala = 108 cm²** .

Aria totala = Aria laterala + Aria bazei $\Rightarrow$ **Aria totala = (108 + 36√3) cm²** .

Volumul $= \frac{\textbf{Aria bazei} \cdot \textbf{inaltimea}}{\mathbf{3}} \Rightarrow$ Volumul $= \frac{36\sqrt{3} \cdot 2\sqrt{6}}{3} \Rightarrow$ **Volumul = 72√2 cm³** .

b) $\angle$**((SAM) ; (SAB))**

Daca SA $\perp$ (SBC) si SB $\subset$ (SBC) $\Rightarrow$ SA $\perp$ SB $\Rightarrow$ **SB ⊥ SA**

$\left.\begin{array}{l} (SAM) \cap (SAB) = SA \\ SM \perp SA ; SM \subset (SAM) \\ SB \perp SA ; SB \subset (SAB) \end{array}\right\} \Rightarrow$ $\angle$ **((SAM) ; (SAB))** = $\angle$ (SM ; SB) = $\angle$**(MSB)**

In ΔSMB dr. avem SM=MB=6 cm $\Rightarrow$ ΔSMB dr. isoscel $\Rightarrow$ **m ∠(MSB) = 45°** $\Rightarrow$ **∠((SAM);(SAB)) = 45°**

c) d(M ; (SAB))

Intre punctul M si planul (SAB) formez piramida MSAB si scriu volumul ei in doua moduri $\Rightarrow$

$\text{Volumul}_{MSAB} = \frac{\text{Aria } \Delta SAB \cdot d(M ; (SAB))}{3} = \frac{\text{Aria } \Delta ABM \cdot SO}{3} \Rightarrow d(M ; (SAB)) = \frac{\text{Aria } \Delta ABM \cdot SO}{\text{Aria } \Delta SAB}$

Aria ΔABM $= \frac{\text{Aria bazei}}{2} = \frac{36\sqrt{3}}{2} = \mathbf{18\sqrt{3}\ cm^2}$; **Aria ΔSAB** $= \frac{SA \cdot SB}{2} = \frac{6\sqrt{2} \cdot 6\sqrt{2}}{2} = \mathbf{36\ cm^2}$

$d(M ; (SAB)) = \frac{18\sqrt{3} \cdot 2\sqrt{6}}{36} = \sqrt{18} = 3\sqrt{2}$ cm $\Rightarrow$ **d(M ; (SAB)) = 3√2 cm** .