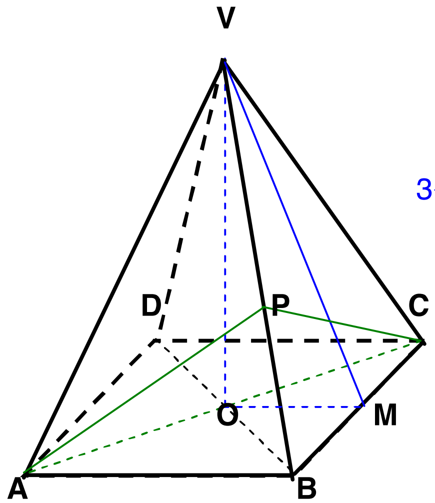
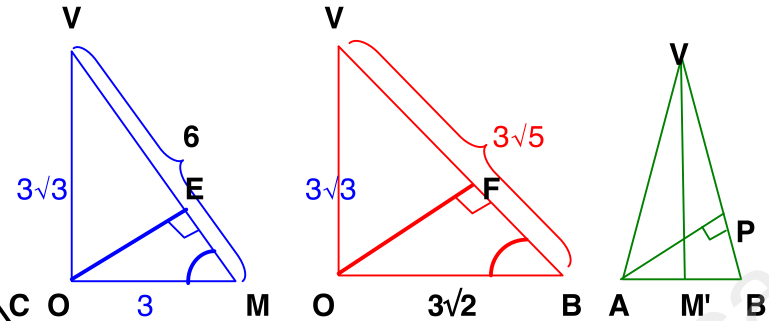
I1. PIRAMIDA PATRULATERA REGULATA - PROBLEME REZOLVATE

1. O piramida patrulatera regulataVABCD are, apotema VM=6cm si diagonala bazei AC=6√2cm. Se cere: a) Aria laterala, aria totala, volumul; b) Distanta de la centrul bazei la o fata laterala si distanta de la centrul bazei la o muchie laterala; c) Unghiul dintre fata laterala si planul bazei; d) Sinusul unghiului dintre muchia laterala si planul bazei; e) Fie un punct P pe muchia VB. Determinati lungimea segmentului BP astfel incit perimetrul ΔAPC sa fie minim.





a) $A_l = \dfrac{P_b \cdot ap}{2}$; $A_t = A_l + A_b$; $V = \dfrac{A_b \cdot h}{3}$

$AC = l\sqrt{2} \Rightarrow l\sqrt{2} = 6\sqrt{2} \Rightarrow l=6 \Rightarrow$ **AB=6cm** ; $OM = \dfrac{AB}{2} = \dfrac{6}{2} \Rightarrow$ **OM=3cm**

In $\Delta VOM, \angle O = 90° \Rightarrow VO^2 = VM^2 - OM^2 = 36 - 9 = 27 \Rightarrow$ **VO=3√3cm**

In$\Delta VOB, \angle O = 90° \Rightarrow VB^2 = VO^2 + OB^2 = 27 + 18 = 45 \Rightarrow$ **VB=3√5cm**

$A_l = \dfrac{4 \cdot 6 \cdot 6}{2} = 72cm^2$; $A_b = l^2 = 6^2 = 36cm^2$; $A_t = 72 + 36 = 108cm^2$; $V = \dfrac{36 \cdot 3\sqrt{3}}{3} = 36\sqrt{3}cm^3$.

b) $(VOM) \perp (VBC)$ - VM = latura comuna $\Rightarrow d(O;(VBC)) = d(O;VM) = OE$ $(OE \perp VM)$

In $\Delta VOM, \angle O = 90° \Rightarrow OE = \dfrac{VO \cdot OM}{VM} = \dfrac{3\sqrt{3} \cdot 3}{6} \Rightarrow$ **$OE = \dfrac{3\sqrt{3}}{2}$ cm**

$OF \perp VB \Rightarrow d(O,VB) = OF$. In $\Delta VOB, \angle O = 90° \Rightarrow OF = \dfrac{VO \cdot OB}{VB} = \dfrac{3\sqrt{3} \cdot 3\sqrt{2}}{3\sqrt{5}} \Rightarrow$ **$OF = \dfrac{3\sqrt{30}}{5}$ cm.**

c) $\left.\begin{array}{l} (VBC) \cap (ABCD) = BC \\ VM \perp BC;\ VM \subset (VBC) \\ OM \perp BC;\ OM \subset (ABCD) \end{array}\right\} \Rightarrow \angle((VBC);(ABCD)) = \angle(VM;OM) = \angle(VMO)$

In $\Delta VOM, \angle O = 90°$, $OM = \dfrac{VM}{2} \Rightarrow m(\angle OVM) = 30° \Rightarrow$ **$m(\angle VMO) = 60°$.**

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d) OB este proiectia lui VB pe planul ABCD $\Rightarrow \angle(VB;(ABCD)) = \angle(VB;OB) = \angle(VBO)$

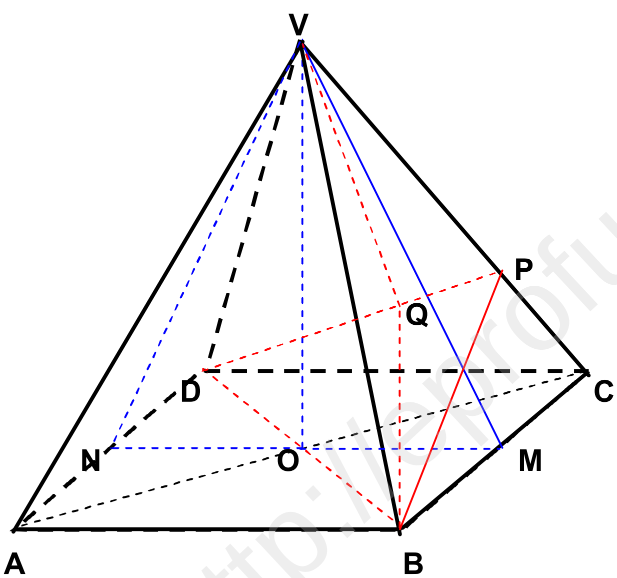
In $\Delta VBO, \angle O=90° \Rightarrow \sin(\angle VBO) = \dfrac{VO}{VB} = \dfrac{3\cdot\sqrt{3}}{3\sqrt{5}} \Rightarrow$ **$\sin(\angle VBO) = \dfrac{\sqrt{15}}{5}$**

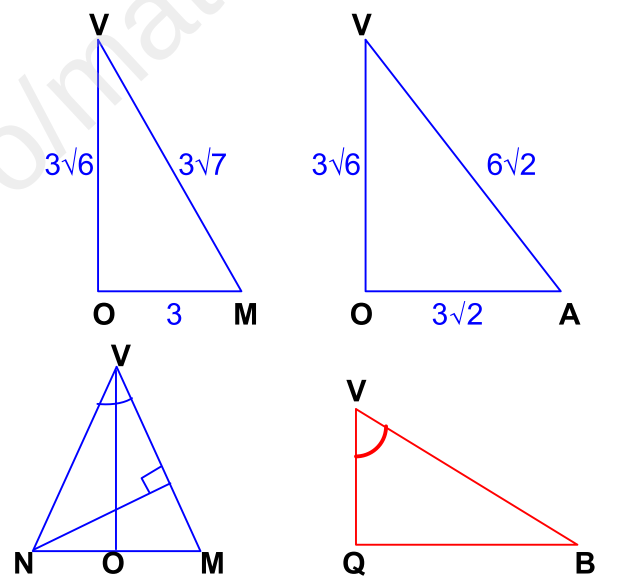
e) ΔAPC este isoscel cu AP=PC. Perimetrul este minim cind AP minim $\Rightarrow AP \perp VB$

In $\Delta VAB \Rightarrow AP\cdot VB = VM'\cdot AB \Rightarrow AP = \dfrac{VM'\cdot AB}{VB} = \dfrac{6\cdot 6}{3\sqrt{5}} = \dfrac{12\sqrt{5}}{5}$ cm

In $\Delta APB, \angle P=90° \Rightarrow PB^2 = AB^2 - AP^2 = 36 - \dfrac{144\cdot 5}{25} = \dfrac{180}{25} \Rightarrow$ **$PB = \dfrac{6\sqrt{5}}{5}$ cm.**

2) Fie o piramida patrulatera regulata de virf V si baza ABCD. Triunghiul VAC este echilateral de arie $18\sqrt{3}$ cm^2. Se cere :a) Aria totala si volumul piramidei VABCD ; b)Distanta de la B la planul (VDC) ; c) Sinusul unghiului dintre muchia VB si planul (VDC); d) sinusul unghiului dintre planele (VAD) si (VBC).





a) Aria $\Delta VAC = \dfrac{VA^2\sqrt{3}}{4} \Rightarrow \dfrac{VA^2\sqrt{3}}{4} = 18\sqrt{3} \Rightarrow VA^2 = \dfrac{4\cdot 18\sqrt{3}}{\sqrt{3}} \Rightarrow$ **VA= $6\sqrt{2}$cm si AC = $6\sqrt{2}$ cm**

$AC=AB\sqrt{2} \Rightarrow AB\sqrt{2} = 6\sqrt{2} \Rightarrow$ **AB=6cm** $\Rightarrow$ **OM=3cm**

In ΔVAB echilateral , VO inaltime $\Rightarrow VO = \dfrac{VA\sqrt{3}}{2} = \dfrac{6\sqrt{2}\cdot\sqrt{3}}{2} \Rightarrow$ **VO=$3\sqrt{6}$cm**

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In $\Delta VOM, \angle O=90° \Rightarrow VM^2 = VO^2 + OM^2 = 54 + 9 = 63 \Rightarrow$ **$VM=3\sqrt{7}$ cm**. **Aria bazei = $AB^2 = 36cm^2$**

Aria laterala = $\dfrac{Pb \cdot ap}{2} = \dfrac{4 \cdot 6 \cdot 3\sqrt{7}}{2}$ = **$36\sqrt{7}cm^2$** ; **Aria totala = Ab + Al** = $(36+36\sqrt{7})$ = **$36(1+\sqrt{7})cm^2$**

Volumul = $\dfrac{Ab \cdot h}{3} = \dfrac{36 \cdot 3\sqrt{6}}{3}$ = **$36\sqrt{6}cm^3$.**

b) Construiesc $BP \perp VC$ si $DP \perp VC$, dar BP si DP $\subset (BPD) \Rightarrow VC \perp (BPD)$

Constuiesc $BQ \perp DP$, dar $BQ \subset (BPD)$ si $VC \perp (BPD) \Rightarrow VC \perp BQ \Rightarrow BQ \perp VC$

Din $BQ \perp DP$ si $BQ \perp VC \Rightarrow BQ \perp (VDC) \Rightarrow$ **$d(B;(VDC)) = BQ$** .

Pentru a calcula mai usor distanta scriem volumul piramidei VBDC in 2 moduri:

$\Rightarrow BQ \cdot Aria\Delta VDC = VO \cdot Aria\Delta BDC \Rightarrow BQ = \dfrac{Aria\Delta BDC \cdot VO}{Aria\ \Delta VDC}$

$Aria\Delta BDC = \dfrac{Aria\ ABCD}{2} = 18cm^2$; $Aria\Delta VDC = Aria\ \Delta VBC = \dfrac{VM \cdot BC}{2} = \dfrac{3\sqrt{7} \cdot 6}{2} = 9\sqrt{7}cm^2$

$BQ = \dfrac{18 \cdot 3\sqrt{6}}{9\sqrt{7}} \Rightarrow$ **$BQ = \dfrac{6\sqrt{42}}{7}$ cm**

c) $BQ \perp (VDC)$ si $V \in (VDC) \Rightarrow$ proiectia lui VB pe (VDC) este VQ $\Rightarrow \angle(VB;(VDQ)) = \angle(VB;VQ) = \angle BVQ$

In ΔVQB, $\angle Q=90° \Rightarrow \sin(\angle BVQ) = \dfrac{QB}{VB} = \dfrac{6\sqrt{42}}{7} \cdot \dfrac{1}{6\sqrt{2}} \Rightarrow$ **$\sin(\angle BVQ) = \dfrac{\sqrt{21}}{7}$**

d) $VM \perp BC$; $VN \perp AD$; $BC \parallel AD \Rightarrow \angle((VBC);(VAD)) = \angle(VM;VN) = \angle(MVN)$

In $\Delta VNM \Rightarrow NN' \cdot VM = VO \cdot NM \Rightarrow NN' = \dfrac{VO \cdot NM}{VM} = \dfrac{3\sqrt{6} \cdot 6}{3\sqrt{7}} = \dfrac{6\sqrt{42}}{7}$ cm

In $\Delta VN'N$, $\angle N'=90° \Rightarrow \sin(\angle N'VN) = \dfrac{NN'}{VN} = \dfrac{6\sqrt{42}}{7} \cdot \dfrac{1}{3\sqrt{7}} \Rightarrow$ **$\sin(\angle N'VN) = \dfrac{2\sqrt{6}}{7}$**

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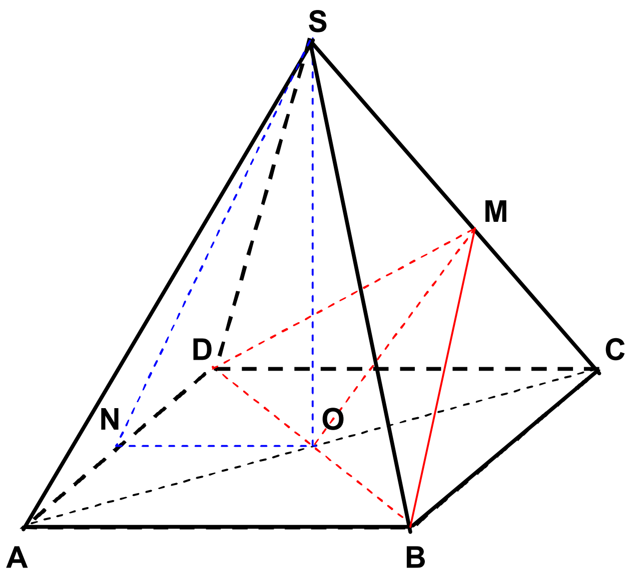
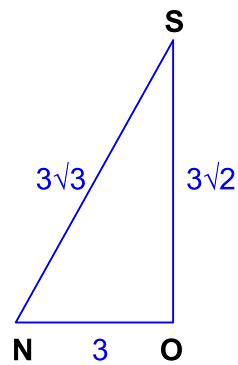
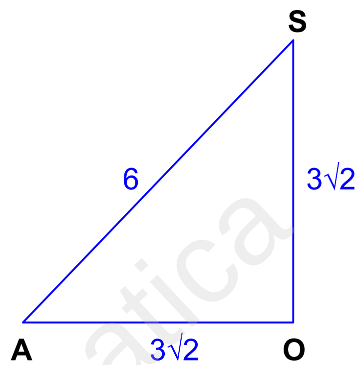
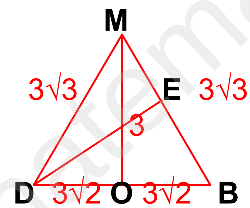
3.Piramida patrulatera regulata SABCD are toate muchiile congruente si aria laterala $36\sqrt{3}$ cm^2.
Se cere:
a) Aria totala si volumul piramidei.
b) Pozitia unui punct M pe muchia VC astfel incat aria ΔMDB sa fie minima.
c) Daca M este la mijlocul muchiei VC calculati unghiul dintre planele (MDB) si (CDB)
d) Sinusul unghiului dintre planele (SBC) si (SAC).
e) Sinusul unghiului dintre doua fețe laterale alaturate

REZOLVARE









a) Deoarece muchiile piamidei sunt congruente $\Rightarrow$ fețele laterale sunt Δ echilaterale

Aria laterala $= 4 \cdot \dfrac{l^2\sqrt{3}}{4} = l^2\sqrt{3} \Rightarrow l^2\sqrt{3} = 36\sqrt{3} \Rightarrow l^2 = 36 \Rightarrow l = 6$ cm $\Rightarrow$ **AB = 6 cm** ; **SA = 6 cm**

Apotema bazei = **ON** $= \dfrac{AB}{2} = \dfrac{6}{2} =$ **3 cm** ; **AC** $= l\sqrt{2} =$ **$6\sqrt{2}$ cm** ; **AO** $= \dfrac{AC}{2} = \dfrac{6\sqrt{2}}{2} =$ **$3\sqrt{2}$ cm**

In ΔSOA dr. $\Rightarrow SO^2 = SA^2 - AO^2 \Rightarrow SO^2 = 6^2 - (3\sqrt{2})^2 = 36 - 18 = 18 \Rightarrow SO = \sqrt{18} \Rightarrow$ **SO = $3\sqrt{2}$ cm**

In ΔSON dr. $\Rightarrow SN^2 = SO^2 + NO^2 \Rightarrow SN^2 = (3\sqrt{2})^2 + 3^2 = 18 + 9 = 27 \Rightarrow SN = \sqrt{27} \Rightarrow$ **SN = $3\sqrt{3}$ cm**

Aria bazei = $l^2 \Rightarrow$ Aria bazei = 36 cm^2

Aria totala = Aria laterala + Aria bazei $= 36\sqrt{3} + 36 =$ **$36(\sqrt{3} + 1)$ cm^2**

Volumul $= \dfrac{\textbf{Aria bazei} \cdot \textbf{inaltimea}}{\textbf{3}} = \dfrac{36 \cdot 3\sqrt{2}}{3} =$ **$36\sqrt{2}$ cm^3**

b) ΔMBD este isoscel $\Rightarrow$ MO este inaltimea triunghiului. Aria ΔMBD $= \dfrac{BD \cdot MO}{2}$

Deoarece BD este constanta $\Rightarrow$ aria este minima daca inaltimea MO este minima

MO este minima daca MO $\perp$ SC

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Deoarece Δ SOC este dreptunghic isoscel $\Rightarrow$ inaltimea MO este si mediana in Δ SOC $\Rightarrow$

M este la mijlocul muchiei SC

c) $\angle$ (**(MDB) ; (CDB)**)

$$\left.\begin{array}{l}(MBD) \cap (CDB) = BD \\ MO \perp BD \; ; \; MO \subset (MBD) \\ CO \perp BD \; ; \; CO \subset (CDB)\end{array}\right\} \Rightarrow \angle\, \mathbf{((MBD)\,;\,(CDB))} = \angle\,(MO\,;\,CO) = \angle\, \mathbf{(MOC)}$$

In ΔMOC dr. in M cu $m\angle(MCO) = 45^\circ \Rightarrow m\,\angle(MOC) = 45^\circ \Rightarrow$ **$m\,\angle$ ((MBD) ; (CDB)) = 45°**

d) sin $\angle$((SBC) ; (SAC))

$$\left.\begin{array}{l}(SBC) \cap (SAC) = SC \\ BM \perp SC \; ; \; BM \subset (SBC) \\ OM \perp SC \; ; \; OM \subset (SAC)\end{array}\right\} \Rightarrow \angle\, \mathbf{((SBC)\,;\,(SAC))} = \angle\,(BM\,;\,OM) = \angle\, \mathbf{(BMO)}$$

In ΔMOB dr. in O $\Rightarrow \sin\angle(BMO) = \dfrac{OB}{MB}$; $OB = OA = 3\sqrt{2}$ cm ; $MB = \dfrac{l\sqrt{3}}{2} = \dfrac{6\sqrt{3}}{2} = 3\sqrt{3}$ cm

$$\sin\angle(BMO) = \frac{3\sqrt{2}}{3\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3} \Rightarrow \mathbf{\sin\angle(BMO) = \frac{\sqrt{6}}{3}}$$

e) sin $\angle$ ((SDC) ; (SBC))

$$\left.\begin{array}{l}(SDC) \cap (SBC) = SC \\ DM \perp SC \; ; \; DM \subset (SDC) \\ BM \perp SC \; ; \; BM \subset (SBC)\end{array}\right\} \Rightarrow \angle\, \mathbf{((SDC)\,;\,(SBC))} = \angle\,(DM\,;\,BM) = \angle\, \mathbf{(DMB)}$$

In ΔDMB isoscel construiesc $DE \perp MB \Rightarrow DE \cdot MB = MO \cdot DB \Rightarrow DE = \dfrac{MO \cdot DB}{MB} \Rightarrow$

$$DE = \frac{3 \cdot 6\sqrt{2}}{3\sqrt{3}} = \frac{6\sqrt{2}}{\sqrt{3}} = \frac{6\sqrt{6}}{3} = 2\sqrt{6} \Rightarrow \mathbf{DE = 2\sqrt{6}\ cm}$$

In ΔMED dr. in E $\Rightarrow \sin\angle(DME) = \dfrac{DE}{DM} = \dfrac{2\sqrt{6}}{3\sqrt{3}} = \dfrac{2\sqrt{18}}{3 \cdot 3} = \dfrac{2 \cdot 3\sqrt{2}}{3 \cdot 3} = \dfrac{2\sqrt{2}}{3} \Rightarrow$

$$\Rightarrow \mathbf{\sin\angle(DMB) = \frac{2\sqrt{2}}{3}}$$

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