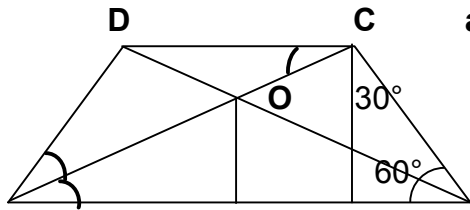


## C2. TRAPEZUL

1) Un trapez isoscel ABCD , are  $AB \parallel CD$ , diagonala AC este bisectoarea unghiului DAB ,  $m(\angle DAB)=60^\circ$  , segmentul care uneste mijloacele diagonalelor are 3cm. Se cere:

a) Aria si perimetrul trapezului ; b) Daca O este intersectia diagonalelor se cere aria  $\triangle BOC$

### REZOLVARE



a) AC bisectoare  $\Rightarrow m(\angle DAC)=m(\angle BAC)$   
 $DC \parallel AB$  , si AC - secanta  $\Rightarrow m(\angle DCA)=m(\angle BAC)$   
 $m(\angle DAC)=m(\angle DCA) \Rightarrow \triangle DAC$  isoscel  $\Rightarrow DA=DC=CB$  }  $\Rightarrow$   
 AB-DC  
 Seg. care uneste mijc. diag. =  $\frac{AB-DC}{2} = EB=3\text{cm}$

In  $\triangle CEB$ ,  $m(\angle E)=90^\circ$ ,  $m(\angle B)=60^\circ \Rightarrow m(\angle C)=30^\circ \Rightarrow$

$\Rightarrow EB = \frac{CB}{2} \Rightarrow CB = 2 \cdot EB = 2 \cdot 3 = 6 \text{ cm} \Rightarrow AD=DC=CB=6 \text{ cm si } AB=12 \text{ cm}$

In  $\triangle CEB$ ,  $m(\angle E)=90^\circ \Rightarrow CE^2=CB^2-EB^2 \Rightarrow CE^2=36-9=27 \Rightarrow CE=3\sqrt{3} \text{ cm}$

Aria ABCD =  $\frac{(AB+DC) \cdot CE}{2} = \frac{(12+6) \cdot 3\sqrt{3}}{2} = 27\sqrt{3} \text{ cm}^2$  ; Perimetrul ABCD =  $3 \cdot 6 + 12 = 30 \text{ cm}$

b) Aria  $\triangle BOC =$  Aria  $\triangle BAC -$  Aria  $\triangle BAO$

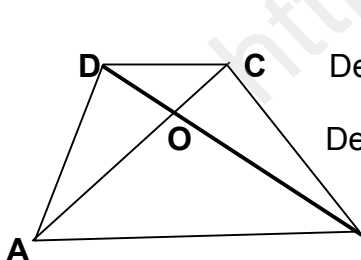
$OF \perp AB \Rightarrow OF \parallel CE \Rightarrow \triangle OFA \sim \triangle CEA \Rightarrow \frac{OF}{CE} = \frac{AF}{AE} \Rightarrow \frac{OF}{3\sqrt{3}} = \frac{6}{9} \Rightarrow OF = \frac{6 \cdot 3\sqrt{3}}{9} \Rightarrow OF = 2\sqrt{3} \text{ cm}$

Aria  $\triangle BAC = \frac{AB \cdot CE}{2} = \frac{12 \cdot 3\sqrt{3}}{2} = 18\sqrt{3} \text{ cm}^2$  ; Aria  $\triangle BAO = \frac{AB \cdot OF}{2} = \frac{12 \cdot 2\sqrt{3}}{2} = 12\sqrt{3} \text{ cm}^2$

Aria  $\triangle BOC = 18\sqrt{3} - 12\sqrt{3} = 6\sqrt{3} \text{ cm}^2$ .

2) Trapezul ABCD ( $AB \parallel CD$ ), are diagonalele perpendiculare, suma bazelor 10 cm, diagonala AC=6cm. Aflati aria trapezului.

### REZOLVARE



Deoarece trapezul este ortodiagonal  $\Rightarrow$  aria =  $\frac{AC \cdot DB}{2}$

Deci pentru calculul ariei trebuie calculata lungimea segmentului BD

Notam baza mare  $AB = B$  si baza mica  $DC = b$

In  $\triangle DOC$ ,  $m(\angle O)=90^\circ \Rightarrow DO^2 = DC^2 - OC^2$  ; In  $\triangle AOB$ ,  $m(\angle O)=90^\circ \Rightarrow OB^2 = AB^2 - AO^2$  (1)

$\triangle DOC \sim \triangle AOB$  (au 2 unghiuri congruente)  $\Rightarrow \frac{DC}{AB} = \frac{DO}{OB} = \frac{CO}{OA}$  , prin proportii derivate  $\Rightarrow$

$$\frac{DC+AB}{AB} = \frac{CO+OA}{OA} \Rightarrow \frac{10}{B} = \frac{6}{OA} \Rightarrow OA = \frac{6 \cdot B}{10} \Rightarrow OA = \frac{3B}{5}$$

$$\frac{DC}{AB+DC} = \frac{CO}{OA+CO} \Rightarrow \frac{b}{10} = \frac{CO}{6} \Rightarrow CO = \frac{6 \cdot b}{10} \Rightarrow CO = \frac{3b}{5}$$

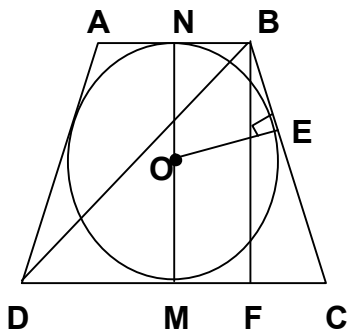
inlocuim in relatiile (1) si obtinem:

$$DO^2 = b^2 - \frac{9b^2}{25} = \frac{16b^2}{25} \Rightarrow DO = \frac{4b}{5}; OB^2 = B^2 - \frac{9B^2}{25} = \frac{16B^2}{25} \Rightarrow OB = \frac{4B}{5}$$

$$DB = DO + OB = \frac{4b}{5} + \frac{4B}{5} = \frac{4(b+B)}{5} = \frac{4 \cdot 10}{5} \Rightarrow DB = 8 \text{ cm} \Rightarrow \text{Aria ABDC} = \frac{6 \cdot 8}{2} = 24 \text{ cm}^2$$

3) Un trapez isoscel circumscris unui cerc cu raza de 4cm , are baza mica 50% din inaltime. Se cere:  
a) Aria si perimetrul trapezului ; b) diagonalele trapezului

### REZOLVARE



a)  $OE \perp BC \Rightarrow BN, BE, CM, CE$  - tangente exterioare la cerc  
 $BN=BE$  si  $CE=CM$  - tangente exterioare la cerc din acelasi punct

$$BC = BE + EC = BN + CM = \frac{AB+CD}{2} \Rightarrow BC = \frac{B+b}{2}$$

In  $\triangle BFC$ ;  $FC = \frac{B-b}{2}$ ;  $BF = 2 \cdot R = 8 \text{ cm}$ ; dar  $b = 50\% \cdot B \Rightarrow b = 4 \text{ cm}$

$$\Rightarrow BC = \frac{B+4}{2}; FC = \frac{B-4}{2}; BF = 8 \text{ cm}$$

$$\text{In } \triangle BFC, m(\angle F) = 90^\circ \Rightarrow BC^2 = BF^2 + FC^2 \Rightarrow \frac{B^2+8B+16}{2} = \frac{B^2-8B+16}{2} + 64 \Rightarrow 16B = 256 \Rightarrow$$

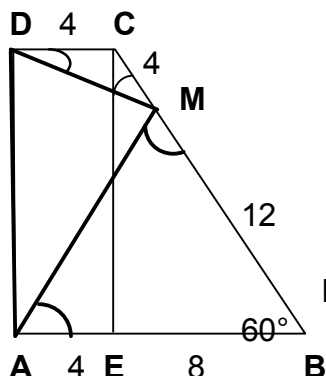
$$B = DC = 16 \text{ cm}$$

$$\text{Aria ABCD} = \frac{(B+b) \cdot h}{2} = \frac{(16+4) \cdot 8}{2} \Rightarrow \text{Aria} = 80 \text{ cm}^2; \text{ Perimetrul ABCD} = 2 \cdot 10 + 16 + 4 = 40 \text{ cm}$$

b) Diagonalele sunt egale. In  $\triangle BFC, m(\angle F) = 90^\circ \Rightarrow BD^2 = BF^2 + FD^2 = 64 + 100 = 164 \Rightarrow BD = 2\sqrt{41} \text{ cm}$ .

4) In trapezul ABCD,  $AB \parallel DC$ ,  $m(\angle A) = m(\angle D) = 90^\circ$ ,  $DC = 4\text{cm}$ ,  $AB = 12\text{cm}$ ,  $BC = 16\text{cm}$ . Pe latura BC se ia un punct M astfel incit  $MB = 3 \cdot MC$ . Se cere perimetrul  $\triangle AMD$ .

### REZOLVARE



$$MB = 3 \cdot MC \Rightarrow BC = 4 \cdot MC \Rightarrow 16 = 4 \cdot MC \Rightarrow MC = 4\text{cm}, MB = 12\text{cm}$$

$$\text{In } \triangle CEB, m(\angle E) = 90^\circ, EB = \frac{BC}{2} \Rightarrow m(\angle C) = 30^\circ \Rightarrow m(\angle B) = 60^\circ$$

$$MB = AB; m(\angle B) = 60^\circ \Rightarrow \triangle ABM \text{ echilateral} \Rightarrow AB = BM = AM = 12\text{cm}$$

$$\text{In } \triangle CEB, m(\angle E) = 90^\circ \Rightarrow CE^2 = BC^2 - BE^2 = 256 - 64 = 192 \Rightarrow CE = 8\sqrt{3}\text{cm} \Rightarrow DA = 8\sqrt{3}\text{cm}$$

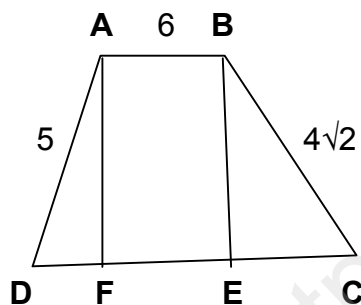
$m(\angle ABC) = 60^\circ \Rightarrow m(\angle BCD) = 120^\circ$ ;  $DC = CM \Rightarrow \triangle DCM$  isoscel  $\Rightarrow m(\angle CMD) = m(\angle CDM) = 30^\circ \Rightarrow$   
 $\Rightarrow m(\angle DMA) = 180^\circ - (60^\circ + 30^\circ) = 90^\circ \Rightarrow \triangle DMA$  dreptunghic in M

$$\text{In } \triangle DMA, m(\angle DMA) = 90^\circ \Rightarrow DM^2 = AD^2 - AM^2 = 192 - 144 = 48 \Rightarrow DM = 4\sqrt{3}\text{cm}$$

$$\text{Perimetrul } \triangle DMA = AM + DA + DM = 12 + 12\sqrt{3} + 4\sqrt{3} = 12(1 + \sqrt{3})\text{ cm.}$$

5) In trapezul ABCD, ( $AB \parallel CD$ ),  $AB = 6\text{cm}$ ,  $BC = 4\sqrt{2}\text{cm}$ ,  $CD = 13\text{cm}$ ,  $DA = 5\text{cm}$ . Se cere aria trapezului

### REZOLVARE



$$(DC + AB) \cdot AF$$

$$\text{Aria ABCD} = \frac{\quad}{2}; \text{trebuie sa calculam inaltimea } AF$$

$$DC = 13; AB = 6 \Rightarrow DF + EC = 13 - 6 = 7\text{cm}$$

$$\text{Notam } DF = x \text{ si } EC = 7 - x$$

$$\text{In } \triangle AFD, \angle F = 90^\circ \Rightarrow AF^2 = 5^2 - x^2$$

$$\text{In } \triangle BEC, \angle E = 90^\circ \Rightarrow BE^2 = (4\sqrt{2})^2 - (7 - x)^2$$

$$\Rightarrow 25 - x^2 = 32 - 49 + 14x - x^2 \Rightarrow 14x = 42 \Rightarrow x = 3 \Rightarrow$$

$$(6 + 13) \cdot 4$$

$$\Rightarrow DF = 3\text{ cm} \Rightarrow AF^2 = 25 - 9 = 16 \Rightarrow AF = 4\text{ cm} \Rightarrow \text{Aria ABCD} = \frac{\quad}{2} = 38\text{ cm}^2.$$