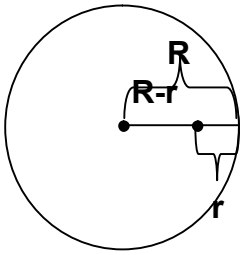
D. CERCUL.

1) Doua cercuri au razele $r_1=(x+3)cm$ si $r_2=(1-3x)cm$, iar distanta dintre centre este 6cm. Pentru ce valoare a lui x , cercurile sunt tangente interior?

REZOLVARE



Cind cercurile sunt tangente interior $\Rightarrow$ distanta $\mathbf{d = R - r \Rightarrow R=r+d}$

r_1 si r_2 trebuie sa fie pozitive $\Rightarrow x>-3$ si $x<1/3 \Rightarrow x\in(-2\ ;\ 1/3)$

Daca $r_1>r_2 \Rightarrow x+3=1-3x+6 \Rightarrow 4x=4 \Rightarrow x=1$

Daca $r_2>r_1 \Rightarrow 1-3x=x+3+6 \Rightarrow -4x=8 \Rightarrow x=-2$

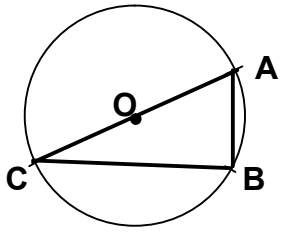
$\Rightarrow$ **x = - 2**

2) Pe cercul de centru O si raza R se iau punctele A, B, C astfel ca arcele AB, BC, CA sa fie proportionale cu 2, 4, 6. Se cere:

a) Masurile arcelor AB, BC, Ca si a unghiurilor ΔABC

b) Daca AC=6cm , aflati perimetrul si aria ΔBC

REZOLVARE



$$\frac{m(AB)}{2}=\frac{m(BC)}{4}=\frac{m(CA)}{6}=\frac{m(AB)+m(BC)+m(CA)}{2+4+6}=\frac{360^\circ}{12}\Rightarrow$$

$$m(AB)=\frac{360\cdot 2}{12}=60^\circ\ ;\ m(BC)=\frac{360\cdot 4}{12}=120^\circ\ ;\ m(CA)=\frac{360\cdot 6}{12}=180^\circ$$

$$m(\angle C)=\frac{1}{2}\cdot m(AB)=30^\circ\ ;\ m(\angle A)=\frac{1}{2}\cdot m(BC)=60^\circ\ ;\ m(\angle B)=\frac{1}{2}\cdot m(CA)=90^\circ$$

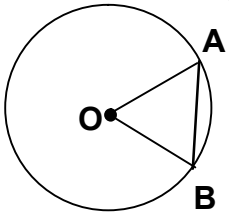
b) In ΔABC, $m(\angle B)=90^\circ$, $m(\angle C)=30^\circ \Rightarrow AB=\frac{AC}{2}=\frac{6}{2}\Rightarrow$ **AB= 3cm**

$\Rightarrow BC^2=AC^2-AB^2=36-9=27 \Rightarrow$ **$BC=3\sqrt{3}$ cm**

Aria $\Delta ABC=\frac{AB\cdot BC}{2}=$ **$\frac{9\sqrt{3}}{2}$ cm^2** , **Perimetrul** $\Delta ABC= 9+3\sqrt{3}=$ **$3(3+\sqrt{3})$ cm.**

3) Pe un cerc de centru O si arie $36\pi\ cm^2$ se ia coarda AB astfel incit lungimea arcului care subintinde coarda AB este 2π cm. Se cere: **a)** $m(\angle AOB)$; **b)** Aria sectorului AOB ; **c)** Ariile segmentelor de cerc determinate de coarda AB.

REZOLVARE



Aria discului $=\pi\cdot R^2 \Rightarrow \pi\cdot R^2=36\cdot\pi \Rightarrow$ **R=6 cm** $\Rightarrow L_{CERC}=2\cdot\pi\cdot R=12\pi$ cm

a) $\angle AOB$ .....2π cm

360° .....12π cm $\Rightarrow \angle AOB=\frac{360^\circ\cdot 2\pi}{12\pi}=$ **60°**

b) 60° .....$Aria_{SECTOR}$

360°..... 36π $\Rightarrow Aria_{SECTOR}=\frac{60^\circ\cdot 36\pi}{360}=$ **$6\pi\ cm^2$.**

c) Aria segmentului (AB 1) = Aria sectorului (OAB) - Aria triunghiului OAB
Aria segmentului (AB 2) = Aria cercului - Aria segmentului (AB 1)

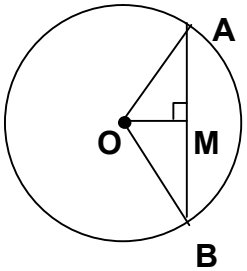
In ΔOAB ; OA=OB = 6cm ; $m(\angle AOB)=60° \Rightarrow \Delta AOB$=ecilateral $\Rightarrow$ Aria $\Delta OAB = \frac{OA^2\sqrt{3}}{4} \Rightarrow$

$\Rightarrow$ Aria$\Delta OAB = 9\sqrt{3}$ cm^2

Aria seg.(AB 1) = $(6\pi - 9\sqrt{3})$ cm^2 ; Aria seg. (AB 2) = $36\pi-6\pi+9\sqrt{3} = (30\pi+9\sqrt{3})$ cm^2.

4) Se da un cerc de centru O si raza R=4cm. La distanta de 2 cm de centru se duce coarda [AB]
Se cere: **a)** $m(\angle AOB)$; **b)** Lungimea sectorului de cerc (AOB) ; **c)** Ariile celor doua segmente de cerc determinate de coarda [AB]

REZOLVARE



a) $OM \perp AB \Rightarrow d(O;AB)=OM=2cm$; ΔOAB isoscel ,OM-mediana si bisectoare

In ΔOMA, $\angle M=90°$, $OM = \frac{OA}{2} \Rightarrow m(\angle OAM)=30° \Rightarrow m(\angle AOM)=60°$

In ΔAOB isoscel $m(\angle AOB) = 2\cdot m(\angle AOM) =$ **120°**

b) Lungimea sectorului OAB = OA+OB+lungimea arcului AB

120° ..... $Lungime_{ARC}$
360°..... $Lungime_{CERC}$ $\Rightarrow Lungime_{ARC}=\frac{120°\cdot Lungime_{CERC}}{360°}$

$Lungime_{CERC}=2\pi R=8\pi cm \Rightarrow Lungime_{ARC} = \frac{120°\cdot 8\pi}{360°} = \frac{8\pi}{3}$ cm

$Aria_{CERC}= \pi R^2=16\pi$ cm^2

Lungime sector OAB= 8 + $\frac{8\pi}{3}$ cm

c) 120°..... $Aria_{SECTOR}$
360°---------- $Aria_{CERC}$ $\Rightarrow Aria_{SECTOR} = \frac{120°\cdot 16\pi}{360°} = \frac{16\pi}{3}$ cm^2

In ΔAMO, $\angle M=90° \Rightarrow AM^2 = AO^2-OM^2 =16-4=12 \Rightarrow AM=2\sqrt{3}$ cm $\Rightarrow$ **AB=4$\sqrt{3}$ cm**

Aria $\Delta OAB= \frac{AB\cdot OM}{2} = 4\sqrt{3}$ cm^2.

Aria segmentului (AB 1)= Aria sectorului OAB - Aria $\Delta OAB = (\frac{16\pi}{3} - 4\sqrt{3})$ cm^2

Aria segmentului (AB 2) = Aria cercului - Aria segm(AB 1) = $16\pi - \frac{16\pi}{3} +4\sqrt{3} = (\frac{32\pi}{3} + 4\sqrt{3})$ cm^2.